

Drinfeld Twists Between Non-Isomorphic Rack Solutions: A Negative Answer to a Question of Ferri

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Abstract

In an extended abstract in Oberwolfach Report 49/2025 (Oberwolfach Rep. 22 (2025), no. 4, 2633–2672; pp. 2645–2649), D. Ferri proposes to classify set-theoretic solutions of the Yang–Baxter equation up to Drinfeld twists, and asks whether two rack solutions on the same set that are related by a Drinfeld twist must coincide. Isomorphic solutions are always related by a twist (Ghobadi, Ferri), so the question is whether such rack solutions must be isomorphic. We show that they need not be. On $X = \mathbb{Z}/5$ the rack solutions $r(a, b) = (2b - a, a)$ and $s(a, b) = (3a + 3b, a)$ of the affine quandles $\text{Aff}(\mathbb{Z}/5, 2)$ and $\text{Aff}(\mathbb{Z}/5, 3)$ are related by an explicit Drinfeld twist, while the two quandles are not isomorphic: the second is commutative and the first is not. Every step is an explicit finite identity, and an exact finite computation confirms all of them. More generally, $\text{Aff}(\mathbb{Z}/p, t)$ and its dual $\text{Aff}(\mathbb{Z}/p, t^{-1})$ are non-isomorphic racks with twist-related rack solutions whenever $-t$ has multiplicative order 3, 4 or 5 modulo the prime p ; this gives such pairs of infinitely many orders. The proofs use Ferri’s criterion in terms of the braid group B_3 , Burnside’s theorem on marks and the finite von Dyck groups. By exact computations, 5 is the smallest order of such a pair, there are 16 such pairs of order 6, and $\text{Aff}(\mathbb{Z}/7, 2)$ is not twist-related to its dual. Status: the definite question answered negatively; the classification programme open. The classification of solutions up to Drinfeld twists stays open, and we do not address it. This is an unrefereed note.

1 Introduction

1.1 The question

A *solution* (of the set-theoretic Yang–Baxter equation) is a set X with a map $r: X^2 \rightarrow X^2$ such that $r_{12}r_{23}r_{12} = r_{23}r_{12}r_{23}$ on X^3 , where $r_{12} = r \times \text{id}_X$ and $r_{23} = \text{id}_X \times r$; maps are composed from right to left. Drinfeld twists of solutions go back to Kulish and Mudrov [KM00]; in the formulation of Ghobadi [Gho21] used by Ferri [Fer25, p. 2646], [Fer26, Definition 2.1], a *Drinfeld twist* for r is a triple (F, Φ, Ψ) of bijections $F: X^2 \rightarrow X^2$ and $\Phi, \Psi: X^3 \rightarrow X^3$ with

$$F_{12}\Psi = F_{23}\Phi, \quad \Psi r_{12} = r_{12}\Psi, \quad \Phi r_{23} = r_{23}\Phi \quad (\text{dt1–dt3})$$

($F_{12} = F \times \text{id}_X$, $F_{23} = \text{id}_X \times F$). Then FrF^{-1} is again a solution, and two solutions r, s on X are *related by a Drinfeld twist*, or *twist-related*, if $s = FrF^{-1}$ for a twist (F, Φ, Ψ) for r . Ferri showed that a bijection F with $FrF^{-1} = s$ extends to such a twist exactly when the permutation representations of the braid group B_3 on X^3 defined by r and s are isomorphic [Fer26, Proposition 2.9], and he proposed to classify all solutions up to Drinfeld twists [Fer25, p. 2648]. A *rack solution* is a left non-degenerate solution whose second component is the first argument [Fer25, p. 2648]; equivalently

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(Section 2), $r(a, b) = (a \triangleright b, a)$ for a rack $(X, \triangleright)$. Since every right non-degenerate solution can be twisted into a rack solution [Fer25, p. 2648], the classification of non-degenerate solutions up to twists reduces to that of racks, and Ferri asks [Fer25, p. 2649]:

Question 1.1 (Ferri). *If (X, r) and (X, s) are rack solutions related by a Drinfeld twist, must they coincide?*

Ferri calls this open and “even hard to check computationally”, and notes that a positive answer would imply that racks classify solutions up to Drinfeld twists [Fer25, p. 2649]. Together with the classification programme, the question forms record OWR-14299580-006 of the UnsolvedMath collection [ula26], where it is listed as open.

The word “coincide” cannot mean equality of maps: isomorphic solutions on the same set are always twist-related ([Fer26, Remark 2.3]; Ferri notes that for set-theoretic solutions this was observed in [Gho21, §2]), so any relabelling that changes the multiplication table of a rack is already a twist that changes the map (Remark 2.4). The reading under which a positive answer would let racks classify solutions up to twists, and the one we answer, is therefore:

(Q) *If two rack solutions on the same set are related by a Drinfeld twist, must the two racks be isomorphic?*

For rack solutions an isomorphism of solutions is the same as an isomorphism of racks (Section 2).

1.2 Results

For a prime p and $t \in \mathbb{F}_p \setminus \{0, 1\}$ let $\text{Aff}(\mathbb{Z}/p, t)$ be the affine (Alexander) quandle on $\mathbb{Z}/p$ with $a \triangleright b = tb + (1 - t)a$ [Joy82, FR92]. Its dual rack, with every left multiplication replaced by its inverse, is $\text{Aff}(\mathbb{Z}/p, t^{-1})$.

Theorem 1.2. *On $X = \mathbb{Z}/5$ let $r(a, b) = (2b - a, a)$ and $s(a, b) = (3a + 3b, a)$, the rack solutions of $\text{Aff}(\mathbb{Z}/5, 2)$ and $\text{Aff}(\mathbb{Z}/5, 3)$. The bijection*

$$F(a, b) = (\ell + 2u^3, \ell + u^3), \quad u = a - b, \quad \ell = 2a + 4b,$$

of X^2 satisfies $\text{Fr}F^{-1} = s$ and extends to a Drinfeld twist (F, Φ, Ψ) for r , given explicitly in Section 3. The quandles $\text{Aff}(\mathbb{Z}/5, 2)$ and $\text{Aff}(\mathbb{Z}/5, 3)$ are not isomorphic. Hence the answer to (Q), and so to Question 1.1, is negative. Status: the definite question answered negatively; the classification programme open.

Theorem 1.3. *Let p be a prime and $t \in \mathbb{F}_p$ with $t \neq 0$, $t^2 \neq 1$ and $1 + t + t^2 \neq 0$, and suppose that p does not divide the order of the group $G_t \leq \text{GL}_2(\mathbb{F}_p)$ of (2). Then $\text{Aff}(\mathbb{Z}/p, t)$ and $\text{Aff}(\mathbb{Z}/p, t^{-1})$ are non-isomorphic racks whose rack solutions are related by a Drinfeld twist. The hypotheses hold whenever $-t$ has multiplicative order 3, 4 or 5, for example for $(p, t) = (5, 2), (7, 3), (11, 2), (13, 4)$, and for every prime $p \equiv 1 \pmod{3}$ with $t = -\omega$, ω a primitive cube root of unity in $\mathbb{F}_p$. In particular there are counterexamples to (Q) of infinitely many orders. Status: the definite question answered negatively; the classification programme open.*

Theorem 1.2 is the case $(p, t) = (5, 2)$ of Theorem 1.3; we prove it separately because its proof consists of finite identities printed in full (Table 1). Section 5 adds facts which rest on exact finite computations and are kept separate from the two theorems: 5 is the smallest order of a counterexample to (Q); the 353 racks of order 6 give 16 unordered pairs of non-isomorphic twist-related racks, 15 of them not pairs of mutually dual racks and 5 of them pairs of quandles; $\text{Aff}(\mathbb{Z}/7, 2)$ is not twist-related to its dual, although every rack of order at most 6 is; and B_4 separates the pair of Theorem 1.2.

1.3 What is known and what is new

The question, the definitions, the criterion in terms of B_3 (Proposition 2.2, which re-proves the direction of [Fer26, Proposition 2.9] that we use) and the remark that the literal reading fails [Fer26, Remark 2.3] are Ferri's; he formulates the twists of Kulish and Mudrov in Ghobadi's language and attributes the observation behind that remark, for set-theoretic solutions, to [Gho21, §2]. Affine quandles and the fact that $\text{Aff}(\mathbb{Z}/p, t)$ determines t (Lemma 4.1) are classical [Joy82, FR92, Nel03]; Burnside's theorem on marks [td87] and the orders of the von Dyck groups [CM80] are classical and are cited, not proved. The numbers of racks and quandles of order at most 6 that we compute agree with the OEIS [OEI26].

Related work, none of which settles (Q): Lebed and Vendramin [LV22] show that all k -derived solutions of a solution (obtained from its reflections k by guitar maps) induce isomorphic braid group actions [LV22, Theorem 1.8, Corollary 1.9]; the non-isomorphic solutions with isomorphic actions in their Examples 1.4 and 1.5 are not pairs of rack solutions in the sense above. Ferri [Fer26] proves that k -derived solutions are Drinfeld twists, shows that the twist class of a permutation solution $(a, b) \mapsto (\lambda(b), \rho(a))$ depends only on $\rho\lambda$ [Fer26, Lemma 3.9], and shows that a group Drinfeld twist between two braided groups of rack type is of his type (I) [Fer26, Remark 4.22]; $\text{Aff}(\mathbb{Z}/5, 2)$ is not a permutation rack, and no braided group is involved in Theorem 1.2. Ghobadi [Gho21] showed that all twists between skew braces come from families of isomorphisms of their additive groups. Martin, Rowell and Torzewska [MRT25], the only work citing [Fer26] that we found, quote [Fer26, Proposition 2.9] for the equivalence between 3-equivalence and the existence of a Drinfeld twist, ask whether 3-equivalence implies ∞ -equivalence for linear braid representations, and sort the 73 solutions on a 3-element set in their Example 7.28.

We found no source showing that two non-isomorphic racks can have twist-related rack solutions; to our knowledge Theorems 1.2 and 1.3 and Section 5 are not in print. A search that finds nothing is not a proof of novelty (see “Scope and priority” at the end).

2 Preliminaries

Definition 2.1. A *rack* is a set X with a binary operation $\triangleright$ such that every left multiplication $L_a = a \triangleright -$ is a bijection and $a \triangleright (b \triangleright c) = (a \triangleright b) \triangleright (a \triangleright c)$ for all a, b, c ; it is a *quandle* if moreover $a \triangleright a = a$. Its *dual* has the left multiplications L_a^{-1} . An *isomorphism* of solutions $(X, r) \rightarrow (Y, s)$ is a bijection $f: X \rightarrow Y$ with $(f \times f)r = s(f \times f)$. Following [Fer25, p. 2646], a bijective solution r (rack solutions are bijective: $r^{-1}(x, a) = (a, L_a^{-1}(x))$) defines an action $\rho_r: \sigma_1 \mapsto r_{12}, \sigma_2 \mapsto r_{23}$ of the braid group $B_3 = \langle \sigma_1, \sigma_2 \mid \sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2 \rangle$ on X^3 , the B_3 -set of r . An isomorphism of B_3 -sets $\rho_r \rightarrow \rho_s$ is a bijection θ of X^3 with $\theta r_{12} = s_{12}\theta$ and $\theta r_{23} = s_{23}\theta$.

For $r(a, b) = (a \triangleright b, a)$ one has $r_{12}r_{23}r_{12}(a, b, c) = ((a \triangleright b) \triangleright (a \triangleright c), a \triangleright b, a)$ and $r_{23}r_{12}r_{23}(a, b, c) = (a \triangleright (b \triangleright c), a \triangleright b, a)$, so r is a solution if and only if $\triangleright$ is left self-distributive. Hence the rack solutions of [Fer25, p. 2648] (left non-degenerate, second component equal to the first argument) are exactly the maps $r(a, b) = (a \triangleright b, a)$ for racks $(X, \triangleright)$. For two such maps, $(f \times f)r = s(f \times f)$ says $f(a \triangleright b) = f(a) \triangleright' f(b)$, so isomorphisms of rack solutions are isomorphisms of racks.

The following is the direction of Ferri's criterion [Fer26, Proposition 2.9] (see also [Fer25, p. 2646]) that we use; we include its short proof.

Proposition 2.2 (Ferri). *Let r be a solution on X and F a bijection of X^2 , and put $s = FrF^{-1}$. If θ is a bijection of X^3 with $\theta r_{12} = s_{12}\theta$ and $\theta r_{23} = s_{23}\theta$, then $\Psi := F_{12}^{-1}\theta$ and $\Phi := F_{23}^{-1}\theta$ make (F, Φ, Ψ) a Drinfeld twist for r . Conversely, for every twist (F, Φ, Ψ) the map $\theta = F_{12}\Psi = F_{23}\Phi$ is such a bijection.*

Proof. $F_{12}\Psi = \theta = F_{23}\Phi$ is (dt1). Since $s_{12} = (FrF^{-1}) \times \text{id} = F_{12}r_{12}F_{12}^{-1}$, we get $\Psi r_{12} = F_{12}^{-1}\theta r_{12} = F_{12}^{-1}s_{12}\theta = r_{12}F_{12}^{-1}\theta = r_{12}\Psi$, which is (dt2); (dt3) is the same computation with $s_{23} = F_{23}r_{23}F_{23}^{-1}$. The converse is the same computation read backwards. $\square$

Corollary 2.3. *Let X be finite and r, s bijective solutions on X . Then r and s are related by a Drinfeld twist if and only if their B_3 -sets on X^3 are isomorphic.*

Proof. One direction is the converse part of Proposition 2.2. Conversely, let θ be an isomorphism of the B_3 -sets. Then $r \times \text{id}_X$ and $s \times \text{id}_X$ are conjugate, so they have the same cycle type. The cycle type of $m \times \text{id}_X$ is that of m with every multiplicity multiplied by $|X|$; hence r and s have the same cycle type on X^2 , $s = FrF^{-1}$ for some bijection F , and Proposition 2.2 applies. $\square$

Remark 2.4 (the literal reading). If f is a bijection of X and $s = (f \times f)r(f \times f)^{-1}$, then $F = f \times f$, $\Psi = \text{id} \times \text{id} \times f$, $\Phi = f \times \text{id} \times \text{id}$ is a Drinfeld twist with $FrF^{-1} = s$ [Fer26, Remark 2.3]: $F_{12}\Psi = f \times f \times f = F_{23}\Phi$, and Ψ, Φ commute with r_{12}, r_{23} because they act on the other leg. For the transposition $f = (0\ 1)$ on $\text{Aff}(\mathbb{Z}/5, 2)$ the twisted solution is a different map, since $f(0 \triangleright 1) = f(2) = 2$ but $f(0) \triangleright f(1) = 1 \triangleright 0 = 4$. So Question 1.1 with “coincide” read as equality of maps has a trivial negative answer, and its content is (Q).

Lemma 2.5 (conventions). *Let $\tau(a, b) = (b, a)$ and $R(a, b, c) = (c, b, a)$. If (F, Φ, Ψ) is a Drinfeld twist for r with $FrF^{-1} = s$, then $(F', \Phi', \Psi') = (\tau F \tau, R \Psi R, R \Phi R)$ is a Drinfeld twist for $\tau r \tau$ with $F'(\tau r \tau)F'^{-1} = \tau s \tau$. For $r(a, b) = (a \triangleright b, a)$ one has $\tau r \tau(a, b) = (b, a \triangleleft b)$ with $a \triangleleft b = b \triangleright a$, which is the form of rack solutions used in [Fer26, Remark 4.22]. Hence the answer to (Q) does not depend on whether racks are written with left or right multiplications.*

Proof. One checks $RF_{23}R = (\tau F \tau)_{12}$, $RF_{12}R = (\tau F \tau)_{23}$, and the same for r . Hence $F'_{12}\Psi' = RF_{23}\Phi R = RF_{12}\Psi R = F'_{23}\Phi'$, $\Psi'(\tau r \tau)_{12} = R\Phi r_{23}R = Rr_{23}\Phi R = (\tau r \tau)_{12}\Psi'$, and likewise for Φ' . Finally $\tau F \tau \tau r \tau \tau F^{-1} \tau = \tau s \tau$. A bijection f is an isomorphism of the racks $\triangleright$ if and only if it is one of the operations $\triangleleft$. $\square$

3 The pair of order five

In this section $X = \mathbb{Z}/5 = \mathbb{F}_5$, $A = \text{Aff}(\mathbb{Z}/5, 2)$ with $a \triangleright b = 2b - a$, and $B = \text{Aff}(\mathbb{Z}/5, 3)$ with $a \triangleright b = 3b - 2a = 3a + 3b$; r and s are their rack solutions. All identities below are identities in $\mathbb{F}_5$.

We prove Theorem 1.2 in six steps. The bijection θ_W of $\mathbb{F}_5^2$ is defined in step (d), D_r, D_s in (1), and the twist is (F, Φ, Ψ) with $\theta = D_s^{-1}(\theta_W \times \text{id})D_r$, $\Psi = F_{12}^{-1}\theta$ and $\Phi = F_{23}^{-1}\theta$.

Proof of Theorem 1.2. (a) *Racks.* For $a \triangleright b = tb + (1 - t)a$ with $t \neq 0$ every L_a is a bijection, $a \triangleright a = a$, and

$$(a \triangleright b) \triangleright (a \triangleright c) = t(tc + (1 - t)a) + (1 - t)(tb + (1 - t)a) = t^2c + t(1 - t)b + (1 - t)a = a \triangleright (b \triangleright c).$$

So A and B are quandles, and r, s are rack solutions (Section 2).

(b) *Non-isomorphism.* In B we have $a \triangleright b = 3(a + b) = b \triangleright a$ for all a, b . In A , $0 \triangleright 1 = 2$ and $1 \triangleright 0 = 4$. A rack isomorphism $f: A \rightarrow B$ would give $f(2) = f(0) \triangleright f(1) = f(1) \triangleright f(0) = f(4)$, which is impossible.

(c) *Linear coordinates on X^3 .* Put $u = a - b$, $v = b - c$, $\ell_r = 3a + b + 2c$, $\ell_s = 2a + b + 3c$ and

$$D_r(a, b, c) = (u, v, \ell_r), \quad D_s(a, b, c) = (u, v, \ell_s). \quad (1)$$

These are bijections, with $D_r^{-1}(u, v, \ell) = (\ell + 3u + 2v, \ell + 2u + 2v, \ell + 2u + v)$ and $D_s^{-1}(u, v, \ell) = (\ell + 4u + 3v, \ell + 3u + 3v, \ell + 3u + 2v)$. Substitution gives, with (u', v', ℓ') the coordinates of the image,

$$\begin{aligned} r_{12}(a, b, c) = (2b - a, a, c) : & \quad u' = 3u, & \quad v' = u + v, & \quad \ell'_r = \ell_r, \\ r_{23}(a, b, c) = (a, 2c - b, b) : & \quad u' = u + 2v, & \quad v' = 3v, & \quad \ell'_r = \ell_r, \\ s_{12}(a, b, c) = (3a + 3b, a, c) : & \quad u' = 2u, & \quad v' = u + v, & \quad \ell'_s = \ell_s, \\ s_{23}(a, b, c) = (a, 3b + 3c, b) : & \quad u' = u + 3v, & \quad v' = 2v, & \quad \ell'_s = \ell_s. \end{aligned}$$

For instance, for r_{12} : $u' = 2b - 2a = 3u$, $v' = a - c = u + v$ and $\ell'_r = 3(2b - a) + a + 2c = 3a + b + 2c$. Thus $D_r r_{12} D_r^{-1} = S_1 \times \text{id}$, $D_r r_{23} D_r^{-1} = S_2 \times \text{id}$, $D_s s_{12} D_s^{-1} = S'_1 \times \text{id}$ and $D_s s_{23} D_s^{-1} = S'_2 \times \text{id}$ with the linear maps

$$S_1(u, v) = (3u, u + v), \quad S_2(u, v) = (u + 2v, 3v), \quad S'_1(u, v) = (2u, u + v), \quad S'_2(u, v) = (u + 3v, 2v)$$

of $W = \mathbb{F}_5^2$. (The forms ℓ_r, ℓ_s are the invariant forms $a + tb + t^2c$ divided by $1 + t + t^2$, for $t = 2$ and $t = 3$; see Lemma 4.2.) So the two B_3 -sets are $W \times \mathbb{F}_5$ with B_3 acting trivially on the factor $\mathbb{F}_5$, and acting on W by $\sigma_i \mapsto S_i$, respectively $\sigma_i \mapsto S'_i$.

(d) *The intertwiner on W .* The six vectors $(1, 0), (0, 1), (1, 1), (1, 2), (1, 3), (1, 4)$ represent the six lines of W . Let T be the map on these representatives given in the column $T(w)$ of Table 1, and define $\theta_W(0) = 0$ and $\theta_W(cw) = c^3 T(w)$ for $c \in \mathbb{F}_5^\times$ and w a representative. The six vectors $T(w)$ lie on six different lines, and $c \mapsto c^3$ is a bijection of $\mathbb{F}_5^\times$, so θ_W is a bijection of W . We claim $\theta_W S_i = S'_i \theta_W$ for $i = 1, 2$. Let w be a representative and $S_i w = dw_2$ with $d \in \mathbb{F}_5^\times$ and w_2 a representative. For $c \in \mathbb{F}_5^\times$ linearity gives $S_i(cw) = (cd)w_2$, so $\theta_W(S_i(cw)) = c^3 d^3 T(w_2)$, while $S'_i(\theta_W(cw)) = c^3 S'_i(T(w))$. Hence the claim is equivalent to $S'_i(T(w)) = d^3 T(w_2)$ for the twelve pairs (i, w) , and these are the twelve rows of Table 1 (the cubes in $\mathbb{F}_5$ are $1^3 = 1, 2^3 = 3, 3^3 = 2, 4^3 = 4$). Consequently $\theta = D_s^{-1}(\theta_W \times \text{id})D_r$ is a bijection of X^3 with $\theta r_{12} = s_{12}\theta$ and $\theta r_{23} = s_{23}\theta$.

(e) *The map F .* On X^2 put $C_r(a, b) = (u, \ell) = (a - b, 2a + 4b)$ and $C_s(a, b) = (a - b, 4a + 2b)$; both have determinant 1, and $C_s^{-1}(u, \ell) = (\ell + 2u, \ell + u)$. From $r(a, b) = (2b - a, a)$ we get $u' = 2b - 2a = 3u$ and $\ell' = 2(2b - a) + 4a = \ell$, so $C_r r C_r^{-1}(u, \ell) = (3u, \ell)$; in the same way $C_s s C_s^{-1}(u, \ell) = (2u, \ell)$. Let $\kappa(u, \ell) = (u^3, \ell)$, a bijection of X^2 . Since $(3u)^3 = 27u^3 = 2u^3$, we have $\kappa(C_r r C_r^{-1})\kappa^{-1} = C_s s C_s^{-1}$, so $F := C_s^{-1}\kappa C_r$ satisfies $FrF^{-1} = s$; explicitly $F(a, b) = (\ell + 2u^3, \ell + u^3)$ with $u = a - b, \ell = 2a + 4b$.

(f) *Conclusion.* By (d), (e) and Proposition 2.2, (F, Φ, Ψ) is a Drinfeld twist for r with $FrF^{-1} = s$; by (b) the racks are not isomorphic. $\square$

4 An infinite family

Let p be a prime, $t \in \mathbb{F}_p \setminus \{0, 1\}$, and let $r_t(a, b) = (tb + (1 - t)a, a)$ be the rack solution of $\text{Aff}(\mathbb{Z}/p, t)$ on $X = \mathbb{F}_p$. Let $G_t \leq \text{GL}_2(\mathbb{F}_p)$ be the group generated by

$$S_1 = S_1(t) = \begin{pmatrix} -t & 0 \\ 1 & 1 \end{pmatrix}, \quad S_2 = S_2(t) = \begin{pmatrix} 1 & t \\ 0 & -t \end{pmatrix}, \quad (2)$$

acting on column vectors $(u, v)^T$, and let W_t be $\mathbb{F}_p^2$ with B_3 acting by $\sigma_i \mapsto S_i$ (one checks $S_1 S_2 S_1 = S_2 S_1 S_2 = \begin{pmatrix} 0 & -t^2 \\ -t & 0 \end{pmatrix}$). Its contragredient W_t^* is $\mathbb{F}_p^2$ with $\sigma_i \mapsto S_i^{-T} = (S_i^T)^{-1}$. For $(p, t) = (5, 2)$ these are the maps S_1, S_2 of Section 3, and $S_i(3) = S'_i$.

Lemma 4.1 (classical; a case of [Nel03]). *For $t, t' \in \mathbb{F}_p \setminus \{0, 1\}$ the racks $\text{Aff}(\mathbb{Z}/p, t)$ and $\text{Aff}(\mathbb{Z}/p, t')$ are isomorphic if and only if $t = t'$.*

Table 1: The twelve identities $S'_i(T(w)) = d^3T(w_2)$, where $S_iw = dw_2$.

i	w	$T(w)$	$S_iw = dw_2$	$S'_i(T(w))$	$d^3T(w_2)$
1	(1, 0)	(1, 0)	(3, 1) = 3(1, 2)	(2, 1)	2(1, 3) = (2, 1)
1	(0, 1)	(0, 1)	(0, 1) = 1(0, 1)	(0, 1)	1(0, 1) = (0, 1)
1	(1, 1)	(1, 2)	(3, 2) = 3(1, 4)	(2, 3)	2(1, 4) = (2, 3)
1	(1, 2)	(1, 3)	(3, 3) = 3(1, 1)	(2, 4)	2(1, 2) = (2, 4)
1	(1, 3)	(2, 2)	(3, 4) = 3(1, 3)	(4, 4)	2(2, 2) = (4, 4)
1	(1, 4)	(1, 4)	(3, 0) = 3(1, 0)	(2, 0)	2(1, 0) = (2, 0)
2	(1, 0)	(1, 0)	(1, 0) = 1(1, 0)	(1, 0)	1(1, 0) = (1, 0)
2	(0, 1)	(0, 1)	(2, 3) = 2(1, 4)	(3, 2)	3(1, 4) = (3, 2)
2	(1, 1)	(1, 2)	(3, 3) = 3(1, 1)	(2, 4)	2(1, 2) = (2, 4)
2	(1, 2)	(1, 3)	(0, 1) = 1(0, 1)	(0, 1)	1(0, 1) = (0, 1)
2	(1, 3)	(2, 2)	(2, 4) = 2(1, 2)	(3, 4)	3(1, 3) = (3, 4)
2	(1, 4)	(1, 4)	(4, 2) = 4(1, 3)	(3, 3)	4(2, 2) = (3, 3)

Proof. Translations $x \mapsto x + c$ are automorphisms of $\text{Aff}(\mathbb{Z}/p, t')$, so we may assume that an isomorphism f satisfies $f(0) = 0$. Then $f(tb) = f(0 \triangleright b) = t'f(b)$ and $f((1-t)a) = f(a \triangleright 0) = (1-t')f(a)$, hence $f((1-t)a + tb) = (1-t')f(a) + t'f(b) = f((1-t)a) + f(tb)$. As $a \mapsto (1-t)a$ and $b \mapsto tb$ are bijections, f is additive, so $f(x) = cx$ with $c \neq 0$, and $f(tb) = t'f(b)$ gives $t = t'$. $\square$

Lemma 4.2 (splitting). *If $1+t+t^2 \neq 0$, the map $D(a, b, c) = (a-b, b-c, (a+tb+t^2c)/(1+t+t^2))$ is a linear bijection of $\mathbb{F}_p^3$ with $D(r_t)_{12}D^{-1} = S_1 \times \text{id}$ and $D(r_t)_{23}D^{-1} = S_2 \times \text{id}$. So the B_3 -set of r_t is isomorphic to $W_t \times \mathbb{F}_p$ with B_3 acting trivially on $\mathbb{F}_p$.*

Proof. We have $(r_t)_{12}(a, b, c) = ((1-t)a + tb, a, c)$ and $(r_t)_{23}(a, b, c) = (a, (1-t)b + tc, b)$. With $u = a - b$, $v = b - c$ the first map gives $u' = -tu$, $v' = u + v$, the second $u' = u + tv$, $v' = -tv$, and both fix the form $a + tb + t^2c$: for instance $(1-t)a + tb + ta + t^2c = a + tb + t^2c$. If $u = v = 0$ then $a = b = c$ and the third coordinate is a ; so D is injective. $\square$

Lemma 4.3 (duality). *If $1+t+t^2 \neq 0$, the matrix $P = \begin{pmatrix} 1+t & -1 \\ -t & 1+t \end{pmatrix}$, of determinant $1+t+t^2$, satisfies $PS_i(t)^{-\top} = S_i(t^{-1})P$ for $i = 1, 2$. Hence P is an isomorphism of B_3 -sets $W_t^* \rightarrow W_{t^{-1}}$.*

Proof. $S_1^{-\top} = \begin{pmatrix} -1/t & 1/t \\ 0 & 1 \end{pmatrix}$ and $S_2^{-\top} = \begin{pmatrix} 1 & 0 \\ 1 & -1/t \end{pmatrix}$, and both $PS_1^{-\top}$ and $S_1(t^{-1})P$ equal $\begin{pmatrix} -(1+t)/t & 1/t \\ 1 & t \end{pmatrix}$, while both $PS_2^{-\top}$ and $S_2(t^{-1})P$ equal $\begin{pmatrix} t & 1/t \\ 1 & -(1+t)/t \end{pmatrix}$. $\square$

Lemma 4.4 (coprime case). *Let a finite group G act linearly on $V = \mathbb{F}_p^n$, and let V^* be the dual space with $(g\varphi)(v) = \varphi(g^{-1}v)$. If p does not divide $|G|$, then V and V^* are isomorphic G -sets.*

Proof sketch. By Burnside's theorem on marks (see [tD87]), two finite G -sets Y, Y' are isomorphic if and only if $|Y^H| = |Y'^H|$ for every subgroup $H \leq G$. Fix H and let I_HV be the span of the vectors $hv - v$. A functional is fixed by H if and only if it vanishes on I_HV , so $\dim(V^*)^H = n - \dim I_HV$. Since $|H|$ is invertible in $\mathbb{F}_p$, the map $e = |H|^{-1} \sum_{h \in H} h$ is an idempotent with image V^H ; it vanishes on I_HV , and $v - ev = |H|^{-1} \sum_{h \in H} (v - hv) \in I_HV$, so its kernel is I_HV and $\dim V^H = n - \dim I_HV$. Hence $|V^H| = |(V^*)^H|$ for all H . $\square$

Proof of Theorem 1.3. Since $1+t^{-1}+t^{-2} = t^{-2}(1+t+t^2) \neq 0$, Lemma 4.2 applies to t and to t^{-1} : the B_3 -sets of r_t and $r_{t^{-1}}$ are $W_t \times \mathbb{F}_p$ and $W_{t^{-1}} \times \mathbb{F}_p$, with trivial action on $\mathbb{F}_p$. By Lemma 4.3, $W_{t^{-1}} \cong W_t^*$. The actions of B_3 on W_t and on W_t^* factor through G_t , acting by g and by $g^{-\top}$ respectively, so Lemma 4.4 for $G = G_t$ gives an isomorphism of B_3 -sets $W_t \cong W_t^*$. Hence the B_3 -sets of r_t and $r_{t^{-1}}$ are isomorphic, and the two rack solutions are twist-related by Corollary 2.3. As $t^2 \neq 1$, the racks are not

isomorphic by Lemma 4.1. An explicit choice of F : in the coordinates $C_t(a, b) = (a - b, (a + tb)/(1 + t))$ one has $C_t r_t C_t^{-1}(u, \ell) = (-tu, \ell)$, so $F = C_{t^{-1}}^{-1} \kappa C_t$ with $\kappa(u, \ell) = (u^{p-2}, \ell)$ satisfies $F r_t F^{-1} = r_{t^{-1}}$; for $p = 5$ this is the map F of Theorem 1.2.

It remains to show that the hypotheses hold when $-t$ has order $k \in \{3, 4, 5\}$; this is Proposition 4.5. For a prime $p \equiv 1 \pmod{3}$ and a primitive cube root of unity $\omega \in \mathbb{F}_p$, the element $t = -\omega$ has $-t = \omega$ of order 3; there are infinitely many such primes (a special case of Dirichlet's theorem). $\square$

Proposition 4.5. *If $-t$ has multiplicative order $k \in \{3, 4, 5\}$ in $\mathbb{F}_p^\times$, then $t \neq 0$, $t^2 \neq 1$, $1 + t + t^2 \neq 0$, and p does not divide $|G_t|$.*

Proof. As $-t \neq \pm 1$, we have $t^2 \neq 1$, and $t \neq 0$. If $1 + t + t^2 = 0$, then t has order 3 and $-t$ has order 6; so $1 + t + t^2 \neq 0$.

(a) S_1 has the eigenvalues 1 and $-t \neq 1$, so S_1 is diagonalisable and $S_1^k = I$; the same holds for S_2 .

(b) W_t is irreducible. The eigenlines of S_1 are $[0 : 1]$ and $[-(1 + t) : 1]$, those of S_2 are $[1 : 0]$ and $[t : -(1 + t)]$. A common eigenline would force $t = 0$ or $(1 + t)^2 = t$, that is $1 + t + t^2 = 0$.

(c) $Z = (S_1 S_2)^3 = (S_1 S_2 S_1)^2$ is the image of the central braid $(\sigma_1 \sigma_2)^3$, so it commutes with S_1 and S_2 . If Z were not scalar, then, commuting with S_1 , which has two distinct eigenvalues, Z would have exactly the eigenlines of S_1 , and likewise exactly those of S_2 , contradicting (b). So Z is scalar.

(d) Let $N = G_t \cap \{\text{scalars}\}$, so that $|N|$ divides $p - 1$, and let $\bar{G} = G_t/N \leq \text{PGL}_2(\mathbb{F}_p)$. With $x = S_1 S_2 S_1$ and $y = S_1 S_2$ we have $S_1 = y^{-1}x$ and $S_2 = S_1^{-1}y$, so $\bar{G}$ is generated by the images of x and y , and by (a) and (c) these satisfy $x^2 = y^3 = (y^{-1}x)^k = 1$. Put $y' = y^{-1}$. Since $(y'x)^k = 1$ is equivalent to $(xy')^k = 1$ (the two elements are conjugate), the group $\langle x, y' \mid x^2, y'^3, (xy')^k \rangle$ is the von Dyck group $D(2, 3, k) = \langle x, y' \mid x^2, y'^3, (xy')^k \rangle$, of order $12k/(6 - k)$, that is 12, 24, 60 for $k = 3, 4, 5$ (the groups A_4, S_4, A_5 ; see [CM80]). So $\bar{G}$ is a quotient of $D(2, 3, k)$, and $|G_t| = |N| |\bar{G}|$ divides $(p - 1) |D(2, 3, k)|$. Finally k divides $p - 1$, so $p \geq 7$, $p \geq 5$, $p \geq 11$ for $k = 3, 4, 5$, and p divides neither $p - 1$ nor 12, 24, 60 respectively. $\square$

For example, $|G_t| = 96, 24, 600, 600, 24, 96$ for $(p, t) = (5, 2), (7, 3), (11, 2), (11, 7), (13, 4), (13, 5)$ (computation).

5 Exact computations

The statements of this section rest on exact finite computations and are not used in Sections 3 and 4. Isomorphism of finite B_n -sets is decided by a complete invariant (the smallest breadth-first code of each orbit over all base points). For every pair reported as twist-related an explicit twist was built (F by matching cycles, θ by matching orbits, Ψ, Φ as in Proposition 2.2) and checked point by point.

Remark 5.1 (duals). The rack $\text{Aff}(\mathbb{Z}/7, 2)$ is not twist-related to its dual $\text{Aff}(\mathbb{Z}/7, 4)$. Let A, B be their B_3 -sets on X^3 , $x_0 = (1, 0, 0)$ and O the orbit of x_0 in A . An equivariant map from O is determined by the image of x_0 ; there are 49 equivariant maps $O \rightarrow A$ but only 7 maps $O \rightarrow B$. An isomorphism $A \rightarrow B$ would give a bijection between the two sets of maps, so $A \not\cong B$, and Corollary 2.3 applies. In the same way $\text{Aff}(\mathbb{Z}/11, 3)$ and $\text{Aff}(\mathbb{Z}/11, 4)$ give 121 and 11 maps, and $\text{Aff}(\mathbb{Z}/13, 2)$ and $\text{Aff}(\mathbb{Z}/13, 7)$ give 169 and 13. The pair $(7, 2)$ lies outside Lemma 4.2, since $1 + 2 + 4 \equiv 0 \pmod{7}$; the argument above does not use that lemma; for $(11, 3)$ and $(13, 2)$, $G_t = \text{GL}_2(\mathbb{F}_p)$. More generally, for $p \in \{5, 7, 11, 13\}$ and all t with $t^2 \neq 1$, the racks $\text{Aff}(\mathbb{Z}/p, t)$ and $\text{Aff}(\mathbb{Z}/p, t^{-1})$ are twist-related exactly when $-t$ has order 3, 4 or 5; the verification runs found the same, one of them up to $p = 31$. So “every finite rack is twist-related to its dual” is false, although by Proposition 5.2 it holds for all racks of order at most 6.

Proposition 5.2 (exact computation). *For $n = 1, \dots, 6$ there are 1, 2, 6, 19, 74, 353 racks of order n up to isomorphism (1, 1, 3, 7, 22, 73 quandles), in agreement with [OEI26].*

- (a) *For $n \leq 4$ no two non-isomorphic racks of order n are twist-related. For $n = 5$ there are 73 twist classes, and the only pair is $\{\text{Aff}(\mathbb{Z}/5, 2), \text{Aff}(\mathbb{Z}/5, 3)\}$, the two racks of order 5 that are not self-dual. So 5 is the smallest order of a counterexample to (Q).*
- (b) *For $n = 6$ there are 338 twist classes: 13 of size 2, one of size 3, the others of size 1. They contain 16 unordered pairs of non-isomorphic twist-related racks. Exactly one of them is a pair of mutually dual racks (the two racks of order 6 that are not self-dual, both quandles); the other 15 are not of the form $\{X, X^{\text{op}}\}$, and all three racks of the class of size 3 are self-dual. Five of the 16 pairs consist of quandles.*
- (c) *Every rack of order at most 6 is twist-related to its dual. With Remark 5.1, 7 is the smallest order of a rack that is not twist-related to its dual.*

All 16 pairs of (b) were certified directly: an explicit twist with (dt1)–(dt3) and $FrF^{-1} = s$ checked pointwise, and non-isomorphism by all 720 bijections. The program with which the results were first obtained certified the 15 pairs formed by the first member of each class and each other member; one of the independent verification runs, and the program `check_note.py` of this note, certified all 16. (The sixteenth pair also follows from two of the others, since twist-relatedness is an equivalence relation [Fer26, Lemma 2.2], [Gho21, Theorem 2.2].) So twist-relatedness of non-isomorphic racks is not only a phenomenon of duality.

Remark 5.3 (B_4 , computation). Let B_4 act on X^4 by $\sigma_i \mapsto r_{i,i+1}$. For the pair of Theorem 1.2 the two B_4 -sets on X^4 (625 points) have the same orbit sizes but are not isomorphic: the orbit of $(0, 0, 0, 1)$ in the B_4 -set of r has 125 points and admits 25 equivariant maps into the B_4 -set of r but only 5 into that of s . For $\text{Aff}(\mathbb{Z}/7, 3)$ and $\text{Aff}(\mathbb{Z}/7, 5)$, in contrast, the B_4 -sets on X^4 are isomorphic (an explicit isomorphism was found and checked). So the twist relation, which is a statement about B_3 , is strictly coarser than isomorphism of the B_n -sets for all n , and B_4 already separates the pair of Theorem 1.2. This concerns permutation representations only.

5.1 Reproducibility

The program `check_note.py` of the reproducibility package (Python standard library only, about ten seconds) recomputes every finite statement of the note: the proof of Theorem 1.2 on all points of X^2 and X^3 (with Table 1 as printed, (dt1)–(dt3) literally, all 120 bijections, and Remark 2.4 and Lemma 2.5 on this example); Lemmas 4.1 to 4.3 on small cases; Proposition 4.5 for all 46 pairs (p, t) with $p \leq 61$ and $-t$ of order 3, 4, 5; explicit twists for the six pairs listed after Proposition 4.5; Remarks 5.1 and 5.3; and Proposition 5.2, by its own enumeration of the 1, 2, 13, 114, 1708, 36538 labelled racks of order $n \leq 6$. The package also contains the programs, with recorded outputs, of the first derivation and of the three independent verification runs.

6 Conclusion

Status: the definite question answered negatively; the classification programme open. Non-isomorphic rack solutions on one set can be twist-related (Theorem 1.2), from the smallest possible order 5 on (Proposition 5.2) and for infinitely many orders (Theorem 1.3), so racks do not classify solutions up to Drinfeld twists in the sense of one rack per twist class. Whether Theorem 1.3 lists all twist-related pairs $\{\text{Aff}(\mathbb{Z}/p, t), \text{Aff}(\mathbb{Z}/p, t^{-1})\}$ is open (Remark 5.1).

Verification. The results were first obtained, with proofs and with programs that certify Theorem 1.2 exhaustively, test the family of Theorem 1.3 and compute the data of Section 5. Three independent verification runs, all AI-assisted, followed; each wrote its own programs and re-ran the original ones, whose recorded outputs were reproduced.

The first run compared the definitions and the question with [Fer25, Fer26] and re-derived Proposition 2.2, Lemma 2.5, every step of the proof of Theorem 1.2 (all rows of Table 1) and Lemmas 4.1 to 4.4 with Proposition 4.5. Its programs found a B_3 -set isomorphism for the pair of Theorem 1.2 by a generic orbit search that does not use the table, checked the closed formulas, built explicit twists for the six pairs (p, t) after Proposition 4.5, reproduced Remark 5.1, and enumerated all racks of order at most 6, certifying all 16 pairs of Proposition 5.2(b). It required three corrections, which were made: the number of pairs of order 6 (16 unordered pairs, of which the first program had certified the 15 formed with the first member of each class), the identification with the von Dyck group in Proposition 4.5(d), and the wording of the status.

The second run read the sources again and wrote its programs from the definitions before reading the original ones: its own twist for the pair of Theorem 1.2, the closed formulas re-implemented from the text, negative controls, its own enumeration of racks of order at most 6 (same numbers, same 16 certified pairs), a test of its isomorphism invariant on random relabellings of all 353 racks of order 6, the scan of Remark 5.1 for $p \leq 31$ with explicit twists, symbolic checks of Lemmas 4.2 and 4.3 over $\mathbb{Q}(t)$, and the computation of Remark 5.3. It required the same correction of the count, the published versions of [Fer26] and [LV22] in the bibliography, and an updated account of the citation search; these were made. When this note was written, the program `check_note.py` was added; it reproduces all finite statements of the note, including Remark 5.3, which the exact program of this note recomputed before the remark was included.

The third run examined the final text of this note line by line before any change. It read again the report [Fer25], pp. 2645–2649, Sections 2 and 3.2 and Remark 4.22 of [Fer26], the parts of [LV22] and [MRT25] cited here and Section 2 of [Gho21], checked the bibliographic data with Crossref, re-derived every proof, recomputed the constants of Section 3 and all rows of Table 1, and repeated the literature search. Its own programs, written from the text, check Theorem 1.2 literally on all points (Table 1 compared entry by entry with the printed one, (dt1)–(dt3), all 120 bijections); verify Lemmas 4.2 and 4.3 over $\mathbb{Q}(t)$, recompute $|G_t|$ for the six pairs after Proposition 4.5, check Proposition 4.5 for all 46 pairs with $p \leq 61$ and obtain the orders 12, 24, 60 by coset enumeration; enumerate the racks of order at most 6 with a separate program in C (same numbers, same 338 twist classes, all 16 pairs certified); and recompute Remarks 5.1 and 5.3, with the scan of Remark 5.1 for $p \leq 19$. It also re-ran the whole package from the archive; all recorded outputs were reproduced, up to running times. It found no mathematical error and required five corrections, which were made: bijectivity in Definition 2.1 and Corollary 2.3, the attribution of the observation of [Fer26, Remark 2.3] to [Gho21], the account of the searches, this description, and two small precisions (Remark 5.1 and the issue date of [Fer25]). The proofs of Theorems 1.2 and 1.3 do not depend on a computation.

Scope and priority. Status: the definite question answered negatively; the classification programme open. The note answers the definite question of [Fer25, p. 2649] negatively, in the reading (Q) and, trivially, in the literal reading; beyond that it contains only Theorem 1.3, Remarks 5.1 and 5.3 and the data of Proposition 5.2. The classification of solutions up to Drinfeld twists, the programme of [Fer25, p. 2648], is not addressed; Ferri notes that this problem has appeared in equivalent forms in several places, which we have not traced. The question, [Fer26, Definition 2.1, Lemma 2.2, Remark 2.3], the notion of rack solution and the criterion [Fer26, Proposition 2.9] are Ferri’s (who credits the definition of twists to [KM00] in the formulation of [Gho21], the observation of his Remark 2.3 for set-theoretic solutions to [Gho21, §2], and the proof of his Lemma 2.2 to [Gho21, Theorem 2.2]);

Proposition 2.2 re-proves the direction of that criterion which we use and is not new. Lemma 4.1 is classical [Nel03], and Burnside’s theorem on marks [tD87] and the orders of the groups $D(2, 3, k)$ [CM80] are cited.

What was read: the report [Fer25], pp. 2645–2649, in the publisher’s open-access PDF; [Fer26] in the arXiv version v2, to which all our theorem numbers refer, namely Section 2, Lemma 3.9 and Remark 4.22. Of the published version of [Fer26] the landing page of the publisher was read: its abstract, and the one recorded by Crossref, agree with the arXiv abstract; the full text was not compared. Of [LV22], in the arXiv version v2, Definition 1.1, Examples 1.2–1.5, Theorem 1.8 and Corollary 1.9; of [Gho21], the abstract, the introduction and Section 2; of [MRT25] (v3), the passages that cite [Fer26] and Example 7.28.

The searches (arXiv API for Ferri’s papers, for “Drinfeld twist” with rack, quandle, set-theoretic solution or braid group, for Lebed or Vendramin with twists, guitar maps and Ghobadi; zbMATH; Crossref; Semantic Scholar; a few web searches) were made in October 2026 by the verification runs and partly repeated for this note; the third run repeated them on 20 October 2026 (ten arXiv queries, zbMATH, Crossref, one web search) with the same outcome. Semantic Scholar lists exactly one paper citing the arXiv version of [Fer26], namely [MRT25], which cites [Fer26, Proposition 2.9] for the equivalence of 3-equivalence with the existence of a Drinfeld twist, asks about linear braid representations, and sorts small set-theoretic solutions in its Example 7.28; it does not settle the question. (This list was obtained in a verification run; the repeated requests, for this note and in the third run, were refused with a rate limit.) OpenAlex lists no work citing the published version of [Fer26]; its search was unavailable (HTTP 429), so the citations of the arXiv version could not be checked there. Crossref records no citations of the published version of [Fer26] and none of the report. Further papers on twists of set-theoretic solutions, skew braces and Hurwitz orbits found by these searches, whose abstracts were read, do not compare two rack solutions up to Drinfeld twists. No printed answer to the question was found in the sources searched; no source we found states that two non-isomorphic racks can have twist-related rack solutions. A search that finds nothing is not a proof of novelty: the phenomenon may be known in the language of Hurwitz actions or of colouring invariants, and this literature was searched by keywords only. We make no priority claim.

AI-assisted tools were used to search the problem corpus and the literature, to find the arguments, to write and run the verification programs, and to help prepare this manuscript. The author has reviewed the proofs, the computations and the sources and is responsible for all claims. The note has not been peer reviewed.

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