

Fixed-Point-Free Representations of Alternating Groups of Twin-Prime Degree: A Partial Answer to a Question of M. Lee

Alper Ferudun*

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Abstract

M. Lee asked at the Oberwolfach workshop “Computational Group Theory” (June 2025) whether, for a prime p such that $p - 2$ is also prime, the alternating group A_p has a faithful irreducible representation over a field of some characteristic $r > 0$ in which all elements of orders p and $p - 2$ act without nonzero fixed vectors, and remarked that this is easily seen to fail for $r > p$. We give a partial answer: yes for $p = 5$, no for $p = 7, 13, 19$ (13 and 19 conditional on published decomposition matrices); open for $p \geq 31$. For $p = 5$ such a representation exists exactly in characteristic 2 ($A_5 \cong \text{SL}_2(4)$ on $\mathbb{F}_4^2$). For $p = 7$ the answer is no in every characteristic, without tables, with explicit certificates in $\mathbb{F}_2 A_7$ and $\mathbb{F}_3 A_7$. For $p = 13$ with $r = 2, 3, 5$ and $p = 19$ with $r = 2, 3, 5, 7$ the answer rests on the decomposition matrices of symmetric groups published by the Modular Atlas project, generated from the character table library of GAP; the other characteristics are excluded without tables. For every such p the question reduces to S_p and to primes $r < p/2$: $r = p$ and $r = p - 2$ are impossible, $r > p$ is Lee’s remark (its short proof also covers characteristic 0), and $p/2 < r < p - 2$ is excluded by a standard Brauer-tree argument, which for a p -cycle alone Zalesski attributes to Thompson. For odd r the question reduces further to the simple modules of S_{p-2} with a fixed-point-free $(p - 2)$ -cycle, a case of an open problem of Zalesski. No printed answer to the question was found in the sources searched; a search that finds nothing is not a proof of novelty. This is an unrefereed note.

1 Introduction

1.1 The question

At the problem session (5 June 2025) of the Oberwolfach workshop “Computational Group Theory” (1–6 June 2025), M. Lee asked the following [DENRD25, p. 1442, Problem 1]. Let p be a prime such that $p - 2$ is also prime. Does A_p have a faithful irreducible representation over a field “in any characteristic $r > 0$ ” in which all elements of orders p and $p - 2$ act fixed-point-freely, that is, without nonzero fixed vectors? Lee added that this is easily shown not to happen if $r > p$.

The problem is record OWR-14299292-011 of the UnsolvedMath collection [ula26], where it is listed as open. The record’s statement begins with “Let p and r be primes” and omits the hypothesis that $p - 2$ is prime; its field with the original wording starts at the question sentence, so the hypothesis is missing there too, and it also contains Problem 2 of the session (a conjecture of Ellis and Harper on derangements), which is a different question. We answer the question of the source, for primes p such that $p - 2$ is prime, and claim nothing for other primes p .

Reading. The remark on $r > p$ shows that the question is asked for each characteristic. We answer it for every pair (p, r) that we treat, which settles it under either reading of “in any characteristic”

*Mercury Software GmbH. Email: alper@mercurycodelab.com. GitHub: <https://github.com/AlperTheKing>.
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(for some r , or for every r). “Irreducible” means irreducible over the given field; by Lemma 2.1 the field plays no role. Faithfulness is automatic: A_p is simple, and the trivial module has fixed vectors. Characteristic 0, projective representations (of $2.A_p$), the fixed-point-freeness of only one of the two classes, and Problem 2 are not the question.

Notation. Throughout, $p \geq 5$ is a prime such that $q := p - 2$ is prime ($p = 5, 7, 13, 19, 31, 43, 61, 73, 103, \dots$). S_p and A_p act on $\{1, \dots, p\}$; $g = (1\ 2 \cdots p)$, $h = (1\ 2 \cdots q)$, and $t = (p-1\ p)$ is the transposition of the two points fixed by h ; $P = \langle g \rangle$ and $H = \langle h \rangle$. The letter r denotes a prime and F an algebraic closure of $\mathbb{F}_r$. For a module V and an element x let $V^x = \{v \in V : xv = v\}$; x is *fixed-point-free* on V if $V^x = 0$. Modules are finite-dimensional.

1.2 Results

Theorem 1.1. *Let $p \geq 5$ be a prime such that $q = p - 2$ is prime.*

- (a) *Reductions (Lemmas 2.1 to 2.4). A representation as in the question exists in characteristic r if and only if some simple FA_p -module has g and h fixed-point-free; it is then realised over a finite field, and “all elements of orders p and $p - 2$ ” may be replaced by “ g and h ”. The characteristics p and q are impossible. For $r \nmid pq$, a simple FS_p -module with Brauer character φ has g and h fixed-point-free if and only if $\varphi(g) = -\varphi(1)/(p-1)$ and $\varphi(h) = -\varphi(1)/(p-3)$, so $(p-1)(p-3)/2$ divides its dimension ($\text{lcm}(p-3, \frac{p-1}{2})$ for A_p), and the question for A_p is equivalent to that for S_p .*
- (b) *The case $r > p$ of Lee’s remark. If $r > p$, and in characteristic 0, no such representation exists (Lemma 3.1).*
- (c) *The case $p/2 < r < p - 2$. If $p \geq 7$ and $p/2 < r < p - 2$, no such representation exists (Lemma 3.2, a standard Brauer-tree argument).*

Consequently, for $p \geq 7$ a representation can exist only in a characteristic $r < p/2$, with the divisibility of (a) (Corollary 3.3).

- (d) $p = 5$: yes, exactly for $r = 2$: $A_5 \cong \text{SL}_2(4)$ acting on $\mathbb{F}_4^2$; over $\mathbb{F}_2$ this is a 4-dimensional irreducible $\mathbb{F}_2 A_5$ -module with endomorphism ring $\mathbb{F}_4$ (Proposition 5.1).
- (e) $p = 7$: no, for every r , by a proof without tables (Proposition 6.2).
- (f) $p = 13$ and $p = 19$: no, for every r . For $p = 13$ with $r \in \{2, 3, 5\}$ and $p = 19$ with $r \in \{2, 3, 5, 7\}$ this is conditional on the decomposition matrices of S_{13} , S_{17} and S_{19} published by the Modular Atlas project [Mod]; for $p = 13$ with $r = 7$ and $p = 19$ with $r \in \{11, 13\}$ it follows from (c), and for $r \in \{p, q\}$ or $r > p$ from (a) and (b) (Proposition 7.1).
- (g) *Transposition twist. Let r be odd, $r \nmid pq$, and $q \geq 8$. If the only simple FS_q -modules on which a q -cycle is fixed-point-free are the natural module N (Brauer character $\pi - 1$, with π the number of fixed points) and $N \otimes \text{sgn}$, then no nonzero FS_p -module has h fixed-point-free (Lemma 4.1).*

Problem 1.2 (open cases). *For every prime $p \geq 31$ such that $p-2$ is prime ($p = 31, 43, 61, 73, 103, \dots$) and every prime $r < p/2$, it is not known whether there is a simple FA_p -module, equivalently a simple FS_p -module, on which g and h act fixed-point-freely.*

All computed cases point to the answer no for every $p \geq 7$, but this is not proved. For odd r , Theorem 1.1(g) reduces the open cases to the classification of the simple FS_{p-2} -modules on which a $(p-2)$ -cycle is fixed-point-free, a case of an open problem of Zalesski [Zal25b, Problem 4] (Remark 4.2); for $r = 2$ no reduction is known to us. The answer that Zalesski expects would give no for all $p \geq 7$ (Remark 8.1). In short, the note gives a *partial answer: yes for $p = 5$, no for $p = 7, 13, 19$ (13 and 19 conditional on published decomposition matrices); open for $p \geq 31$.*

1.3 Credits and limits

The question and the remark on $r > p$ are due to Lee [DENRD25]; Lemma 3.1 is that remark, with its short proof for completeness. Lemma 3.2 is a standard Brauer-tree argument, not new: for a p -cycle alone, Zalesski [Zal25b, p. 6] attributes the case $\ell > n/2$ of Problem 4 of that survey to Thompson [Tho08, Lemma 3.2], which we did not read and cite as described by Zalesski. The other lemmas use standard tools; the transposition twist (Lemma 4.1) was not found in print in this form, but we claim no priority for it. The note contributes the explicit answers for $p = 5, 7, 13, 19$ (for 13 and 19 only modulo the published tables), the reduction of Lemma 4.1, and the exclusion of all $r \geq p/2$ assembled for this question. Modules with fixed-point-free elements of prime order occur in recent work on elusive groups [CLM⁺25] and on the Polycirculant Conjecture [FL26]; these papers, like [Zal25a] and [Vel24] (other eigenvalue conditions, characteristic 0), do not answer the question. The searches are described in “Scope and priority” (Section 9).

2 Reductions

Lemma 2.1 (the field). *Let G be a finite group, $x_1, \dots, x_m \in G$, and k a field of characteristic $r > 0$. If some irreducible kG -module V has $V^{x_i} = 0$ for all i , then so has some simple FG -module W , and $W \cong W_0 \otimes_{\mathbb{F}_r} F$ for an absolutely irreducible $\mathbb{F}_r G$ -module W_0 . Conversely, such a W_0 is an irreducible $\mathbb{F}_r G$ -module with $W_0^{x_i} = 0$.*

Proof. V is finite-dimensional, being cyclic. Let $K \supseteq k$ be algebraically closed and $F' \subseteq K$ the algebraic closure of $\mathbb{F}_r$, so $F' \cong F$. Kernels of linear maps commute with field extension, so $(V \otimes_k K)^{x_i} = 0$, and the same holds for a simple KG -submodule W' of $V \otimes_k K$. By Wedderburn’s theorem $F'G/J(F'G) \cong \prod_j M_{n_j}(F')$; as $J(F'G) \otimes K$ is a nilpotent ideal of KG with semisimple quotient $\prod_j M_{n_j}(K)$, we have $W' \cong W \otimes_{F'} K$ for a simple $F'G$ -module W , and $\dim W^{x_i} = \dim W'^{x_i} = 0$. The finitely many matrix entries of the elements of G on W lie in some $\mathbb{F}_{r^a}$, which gives W_0 . The converse is clear. $\square$

Lemma 2.2 (the two classes). *The elements of order p of A_p are the p -cycles, and the subgroups of order p are conjugate to P . The elements of order q are the q -cycles, which form one class of A_p . Hence all elements of orders p and q are fixed-point-free on an A_p -module V if and only if $V^g = V^h = 0$.*

Proof. An element of prime order ℓ is a product of disjoint ℓ -cycles; two q -cycles would need $2q > p$ points, and odd cycles are even permutations. The subgroups of order p are Sylow subgroups. The q -cycles form one class of S_p , and the centraliser of h contains the odd permutation t . Finally $V^x = V^{\langle x \rangle}$ and $V^{xyx^{-1}} = yV^x$. $\square$

Lemma 2.3 ($r = p$ or $r = q$). *If $r = p$ (resp. $r = q$), then g (resp. h) has a nonzero fixed vector on every nonzero module in characteristic r .*

Proof. If x has order r , then $(x - 1)^r = x^r - 1 = 0$, so $x - 1$ is nilpotent. $\square$

From now on $r \nmid pq$. Brauer characters are taken with respect to a fixed isomorphism between the roots of unity of order prime to r in F and in $\mathbb{C}$ (see, for instance, [Nav98]). For an r -regular element x of order m and a module V with Brauer character β , x acts semisimply, $\beta(x)$ is the sum of the lifted eigenvalues, and $\beta(x^j) = \sigma_j(\beta(x))$ for j prime to m , where σ_j is the automorphism $\zeta_m \mapsto \zeta_m^j$ of $\mathbb{Q}(\zeta_m)$. By the orthogonality relations of $\langle x \rangle$,

$$\dim V^x = \frac{1}{m} \sum_{j=0}^{m-1} \beta(x^j), \quad (1)$$

so fixed-space dimensions of r -regular elements are additive along composition series.

Lemma 2.4 (criterion; reduction to S_p). *Let $r \nmid pq$.*

- (i) *Let D be a simple FS_p -module with Brauer character φ . Then φ is integer-valued, and $D^g = D^h = 0$ if and only if $\varphi(g) = -\varphi(1)/(p-1)$ and $\varphi(h) = -\varphi(1)/(p-3)$. In that case $(p-1)(p-3)/2 = \text{lcm}(p-1, p-3)$ divides $\dim D$.*
- (ii) *Let W be a simple FA_p -module with Brauer character β , and $g' = g^n$ with n a non-square modulo p . Then $W^g = W^h = 0$ if and only if $\beta(h) = -\beta(1)/(p-3)$ and $\beta(g) + \beta(g') = -2\beta(1)/(p-1)$; in that case $\text{lcm}(p-3, \frac{p-1}{2})$ divides $\dim W$.*
- (iii) *Every simple FA_p -module W is a direct summand of $D|_{A_p}$ for a simple FS_p -module D , and every simple FS_p -module D arises in this way. For such W and D , $W^g = 0$ if and only if $D^g = 0$, and $W^h = 0$ if and only if $D^h = 0$.*

Hence the question for A_p is equivalent to the question for S_p .

Proof. (i) An element x of S_p is conjugate to x^j for j prime to its order (same cycle type). So $\varphi(x) = \varphi(x^j) = \sigma_j(\varphi(x))$ for all σ_j , and $\varphi(x)$ is a rational algebraic integer, i.e. an integer. Using only $\varphi(g^j) = \varphi(g)$ ($1 \leq j < p$) and $\varphi(h^j) = \varphi(h)$ ($1 \leq j < q$), (1) gives

$$\dim D^g = \frac{\varphi(1) + (p-1)\varphi(g)}{p}, \quad \dim D^h = \frac{\varphi(1) + (p-3)\varphi(h)}{p-2},$$

which is the criterion; as $\varphi(g), \varphi(h) \in \mathbb{Z}$, both $p-1$ and $p-3$ divide $\varphi(1)$.

(ii) The h^j are A_p -conjugate to h (Lemma 2.2), so $\beta(h) \in \mathbb{Z}$ as in (i). Identify $\{1, \dots, p\}$ with $\mathbb{Z}/p$, with g the map $x \mapsto x+1$. The map $m_j: x \mapsto jx$ satisfies $m_j g m_j^{-1} = g^j$, and the permutations conjugating g to g^j form the coset $m_j \langle g \rangle$, all of sign $\text{sgn}(m_j)$. The homomorphism $j \mapsto \text{sgn}(m_j)$ from $(\mathbb{Z}/p)^\times$ to $\{\pm 1\}$ is onto (for a generator c , m_c is a cycle of even length $p-1$), so its kernel is the group of squares (Zolotarev's lemma): g^j is A_p -conjugate to g if j is a square and to g' otherwise. Now (1) gives $\dim W^g = (\beta(1) + \frac{p-1}{2}(\beta(g) + \beta(g')))/p$ and $\dim W^h = (\beta(1) + (p-3)\beta(h))/(p-2)$. Each σ_j fixes or interchanges $\beta(g)$ and $\beta(g')$, so $\beta(g) + \beta(g') \in \mathbb{Z}$.

(iii) Let D be a simple quotient of $\text{Ind}_{A_p}^{S_p} W$; by Frobenius reciprocity $\text{Hom}_{A_p}(W, D|_{A_p}) \cong \text{Hom}_{S_p}(\text{Ind } W, D) \neq 0$. By Clifford's theorem, valid in every characteristic, $D|_{A_p}$ is semisimple, a sum of copies of W and of its conjugate W^t ; so W is a summand, and every D arises in this way. The fixed space of x on W^t is that of txt^{-1} on W ; as tgt^{-1} generates a subgroup conjugate to P and tht^{-1} is conjugate to h in A_p , W^t has the same fixed-space dimensions as W . So $\dim D^g$ and $\dim D^h$ are positive multiples of $\dim W^g$ and $\dim W^h$. $\square$

3 Large characteristics

We use standard facts on S_p , see [Jam78]. By the Murnaghan–Nakayama rule, $\chi^\lambda(g)$ is a signed sum over the rim hooks of length p of $\lambda \vdash p$; such a rim hook is the whole diagram, so

$$\chi^{(p-k, 1^k)}(g) = (-1)^k, \quad \chi^\lambda(g) = 0 \text{ if } \lambda \text{ is not a hook.} \quad (2)$$

Two characters lie in the same r -block if and only if their partitions have the same r -core (Nakayama's conjecture, proved by Brauer and Robinson); the weight w of a block with core κ is given by $p = |\kappa| + rw$.

Lemma 3.1 ($r > p$; Lee's remark). *If $r > p$, or in characteristic 0, no simple module of S_p or of A_p has g and h fixed-point-free.*

Proof. For $r > p$ the algebra FS_p is semisimple and the Brauer characters of the simple modules are the ordinary characters χ^λ . By (2), $|\varphi(g)| \leq 1$. If g is fixed-point-free, then $\varphi(g) = -\varphi(1)/(p-1) \neq 0$ (Lemma 2.4(i)), so $\varphi(g) = -1$ and $\dim D = p-1$, which for $p \geq 7$ is not divisible by $(p-1)(p-3)/2 > p-1$. For $p = 5$ the characters of degree 4 are $\chi^{(4,1)}$ and $\chi^{(2,1,1,1)} = \text{sgn} \cdot \chi^{(4,1)}$, with value $+1$ at the 3-cycle h , whereas Lemma 2.4(i) requires -2 . Lemma 2.4(iii) gives the statement for A_p . In characteristic 0 the same argument applies over an algebraic closure, to which one reduces as in Lemma 2.1. $\square$

Lemma 3.2 ($p/2 < r < p-2$). *Let $p \geq 7$ and let r be a prime with $p/2 < r < p-2$. Every simple FS_p -module on which g is fixed-point-free has dimension at most $2(p-1)$. Hence no simple FS_p -module, and no simple FA_p -module, has g and h fixed-point-free.*

Proof. As $p < 2r$, the Sylow r -subgroups of S_p have order r and every block has weight 0 or 1. A block of weight 0 has defect zero: it contains one ordinary character, whose restriction to the r -regular elements is irreducible, and $|\varphi(g)| \leq 1$ by (2).

A block of weight 1 has a cyclic defect group of order r . By Brauer's theory of blocks of defect one [Bra42] (and Dade's theory of blocks with cyclic defect groups [Dad66]; see [Fei82, Ch. VII]) there is a tree T , the Brauer tree, whose edges are the irreducible Brauer characters of the block and whose vertices are its ordinary characters (one vertex may carry several exceptional characters, which agree on r -regular elements; we choose one), such that the restriction $\hat{\chi}_v$ to the r -regular elements of the character of a vertex v is the sum of the Brauer characters of the edges at v . For an edge $e = \{u, u'\}$ let T_1 be the component of $T \setminus e$ containing u . Then

$$\varphi_e = \sum_{v \in T_1} (-1)^{d(u,v)} \hat{\chi}_v, \quad (3)$$

where d is the distance in T : an edge inside T_1 joins two vertices whose distances from u differ by one, so it occurs twice with opposite signs, and e is the only edge with exactly one end in T_1 . So $\varphi_e(g)$ is a signed sum of values at g of distinct ordinary characters of the block, and only hooks contribute, with ± 1 .

A hook $(A, 1^B)$ of p of weight one loses one r -rim hook to reach its core. A rim hook containing the corner box would be the whole diagram, of size $p \neq r$; so the rim hook lies in the arm or in the leg, and the core is $(A-r, 1^B)$ or $(A, 1^{B-r})$. Hence a block of weight one containing a hook has a hook $\kappa = (a, 1^b)$ as its core, of size $a+b = p-r \geq 2$, and its hooks are exactly $(a+r, 1^b)$ and $(a, 1^{b+r})$, which both have the core κ . Therefore $|\varphi_e(g)| \leq 2$ for every irreducible Brauer character.

If g is fixed-point-free on D , then $\dim D = (p-1)|\varphi(g)| \leq 2(p-1)$ by Lemma 2.4(i). If h is also fixed-point-free, then $(p-1)(p-3)/2 \leq 2(p-1)$, so $p \leq 7$; and for $p = 7$ there is no prime r with $3.5 < r < 5$. Lemma 2.4(iii) gives the statement for A_p . $\square$

This is a standard Brauer-tree argument. For a p -cycle alone, the case $\ell > n/2$ (ℓ the characteristic) of Problem 4 of [Zal25b] is attributed there to Thompson [Tho08, Lemma 3.2]. The hook count is checked by `check_note.py` for all twin primes $p \leq 200$ (Section 9).

Corollary 3.3. *Let $p \geq 7$. If a simple FA_p -module W with g and h fixed-point-free exists, then $r < p/2$, the simple FS_p -modules above W have dimension divisible by $(p-1)(p-3)/2$, and $\dim W$ is divisible by $\text{lcm}(p-3, \frac{p-1}{2})$.*

Proof. Lemmas 2.3, 3.1 and 3.2 exclude $r \in \{p, q\}$, $r > p$ and $p/2 < r < p-2$, and $p-1$ is not prime; the rest is Lemma 2.4. $\square$

4 The transposition twist

Let $S_q \leq S_p$ act on $\{1, \dots, q\}$, and for $z \in S_q$ let $\pi(z)$ be its number of fixed points there. If $r \nmid q$, the permutation module F^q is the sum of the trivial module and the augmentation submodule N , the *natural module*, with Brauer character $\pi - 1$; let $N' = N \otimes \text{sgn}$.

Lemma 4.1 (transposition twist). *Let r be odd, $r \nmid pq$, and $q \geq 8$. Suppose that every simple FS_q -module on which a q -cycle is fixed-point-free is isomorphic to N or N' . Then no nonzero FS_p -module has h fixed-point-free.*

Proof. Let $D \neq 0$ with $D^h = 0$. Fixed-space dimensions of h are additive and nonnegative, so the composition factors of $D|_{S_q}$ are N and N' , with multiplicities m and m' say, and for r -regular $z \in S_q$

$$\beta_D(z) = (m + m' \text{sgn } z)(\pi(z) - 1). \quad (4)$$

Let $Y = S_q \times \langle t \rangle$. As r is odd, the simple FY -modules are the $X \boxtimes \eta$ with X simple for S_q and $\eta(t) = \pm 1$. A composition series of $D|_Y$ restricts to one of $D|_{S_q}$, so the factors of $D|_Y$ are $N \boxtimes \eta$ and $N' \boxtimes \eta$, with multiplicities u_η, u'_η ; put $u = u_1 - u_{-1}$ and $u' = u'_1 - u'_{-1}$. For r -regular $y \in S_q$, the element yt is r -regular and

$$\beta_D(yt) = (u + u' \text{sgn } y)(\pi(y) - 1). \quad (5)$$

If $\pi(y) \geq 2$, let $z \in S_q$ have the cycle type of y with two fixed points replaced by a 2-cycle. Then yt and z have the same cycle type in S_p , so $\beta_D(yt) = \beta_D(z)$, and $\pi(z) = \pi(y) - 2$, $\text{sgn } z = -\text{sgn } y$. With $\sigma = \text{sgn } y$ and $\pi = \pi(y)$, (4) and (5) give

$$(u + u' \sigma)(\pi - 1) = (m - m' \sigma)(\pi - 3). \quad (6)$$

Take $y = 1, (1\ 2)(3\ 4)$ ($\sigma = 1$, $\pi = q, q - 4$) and $y = (1\ 2), (1\ 2)(3\ 4)(5\ 6)$ ($\sigma = -1$, $\pi = q - 2, q - 6$); they are r -regular, and $\pi \geq 2$ as $q \geq 8$. Then

$$\begin{aligned} (u + u')(q - 1) &= (m - m')(q - 3), & (u + u')(q - 5) &= (m - m')(q - 7), \\ (u - u')(q - 3) &= (m + m')(q - 5), & (u - u')(q - 7) &= (m + m')(q - 9). \end{aligned}$$

Eliminating $u + u'$ and $u - u'$, with $(q - 3)(q - 5) - (q - 1)(q - 7) = 8$ and $(q - 5)(q - 7) - (q - 3)(q - 9) = 8$, gives $8(m - m') = 8(m + m') = 0$. So $m = m' = 0$ and $D = 0$, a contradiction. $\square$

Remark 4.2. (a) The program `twist_test.py` runs the same argument with the actual list of simple FS_q -modules on which the q -cycle is fixed-point-free, read from the published tables (without assuming that it is $\{N, N'\}$), and with all r -regular y with $\pi(y) \geq 2$.

(b) For odd r the open cases are thus reduced to the classification of the simple FS_q -modules on which a q -cycle x is fixed-point-free. Then the minimal polynomial of x divides $(X^q - 1)/(X - 1)$, so its degree is less than q ; and by Clifford's theorem, as in Lemma 2.4(iii), x is fixed-point-free on a simple FS_q -module if and only if it is on a simple FA_q -module below it. So this classification is a case of Zaleski's Problem 4 [Zal25b, p. 6]: for the alternating group of prime degree n , an element of order n and an algebraically closed field of characteristic $\ell \notin \{0, n\}$, determine the irreducible representations in which the minimal polynomial of that element has degree less than n . The problem is open there; Zaleski expects these representations to have dimension less than n .

(c) For $r = 2$ the lemma does not apply (t is 2-singular), and no reduction for $r = 2$ is known to us.

5 The case $p = 5$

Proposition 5.1. *For $p = 5$ a representation as in the question exists exactly in characteristic 2. The natural module $V = \mathbb{F}_4^2$ of $\mathrm{SL}_2(4) \cong A_5$ is a faithful irreducible $\mathbb{F}_4 A_5$ -module on which all elements of orders 5 and 3 are fixed-point-free; over $\mathbb{F}_2$ it is a 4-dimensional irreducible $\mathbb{F}_2 A_5$ -module with endomorphism ring $\mathbb{F}_4$.*

Proof. The characteristics 3 and 5 are excluded by Lemma 2.3, and $r > 5$ by Lemma 3.1. In $\mathrm{SL}_2(4)$ an element with eigenvalue 1 has both eigenvalues 1, so $(x - 1)^2 = 0$ and $x^2 = 1$ in characteristic 2; hence elements of order 3 or 5 are fixed-point-free on V . An invariant line would be spanned by an eigenvector of an element of order 5, whose eigenvalues are primitive fifth roots of unity, not in $\mathbb{F}_4$; so V is irreducible, and faithful as A_5 is simple. Over $\mathbb{F}_2$, let $0 \neq U \subsetneq V$ be a submodule and $w \in \mathbb{F}_4 \setminus \mathbb{F}_2$. Then $U + wU$ and $U \cap wU$ are $\mathbb{F}_4 A_5$ -submodules (if $x = wu \in U$ with $u \in U$, then $wx = x + u \in U$), so $U \oplus wU = V$ and $\dim_{\mathbb{F}_2} U = 2$; but A_5 acts trivially on U via $\mathrm{GL}_2(2) \cong S_3$, which contradicts $U^x = 0$. The endomorphism ring is a finite field $E \supseteq \mathbb{F}_4$, and $E \neq \mathbb{F}_{16}$ since otherwise V would be one-dimensional over E and A_5 abelian. $\square$

Certificate. `p5_sl24_module.py` and part A of `check_note.py` take $a = (01)(23)$, $b = (024)$ in A_5 and $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} \omega & 0 \\ \omega & \omega^2 \end{pmatrix}$ over $\mathbb{F}_4 = \mathbb{F}_2(\omega)$, and check that $\langle (a, A), (b, B) \rangle \leq A_5 \times \mathrm{SL}_2(4)$ has 60 elements, with 60 distinct even first and 60 distinct second coordinates, so that it is the graph of an isomorphism; that elements of cycle types (5) and (3, 1, 1) have no nonzero fixed vector; and that no line is invariant. *Consistency.* The simple FS_5 -module above $V \otimes F$ and its Galois conjugate has dimension $4 = (p-1)(p-3)/2$ and, with ζ a primitive fifth root of unity, $\varphi(g) = \zeta + \zeta^{-1} + \zeta^2 + \zeta^{-2} = -1$ and $\varphi(h) = -2$, as Lemma 2.4(i) requires.

6 The case $p = 7$

For a subgroup K let $s_K = \sum_{x \in K} x$. On every module V , $s_K V \subseteq V^K$, since $ks_K = s_K$ for $k \in K$.

Lemma 6.1 (ideal criterion). *Let $G = A_p$, $r \nmid pq$, and $I = \mathbb{F}_r G s_P \mathbb{F}_r G + \mathbb{F}_r G s_H \mathbb{F}_r G$. There is a simple FG -module with g and h fixed-point-free if and only if $\dim_{\mathbb{F}_r} I < |G|$. The ideal I is spanned by the elements zs_K , the indicator vectors of the left cosets zK of the conjugates K of P and H .*

Proof. As $r \nmid |K|$, $s_K/|K|$ acts as a projection onto V^K . If W is simple with $W^g = W^h = 0$, then $(I \otimes F)W = 0$, so $I \otimes F \neq FG$. Conversely, if $I \neq \mathbb{F}_r G$, a simple module of $FG/(I \otimes F)$ is a simple FG -module annihilated by s_P and s_H . Finally $xs_{Ky} = (xy)s_{y^{-1}Ky}$. $\square$

Proposition 6.2. *For $p = 7$ no representation as in the question exists, in any characteristic. More precisely, for $r \in \{2, 3\}$ the identity of $\mathbb{F}_r A_7$ is an explicit combination*

$$1 = \sum_j c_j z_j s_{K_j} \quad (c_j \in \mathbb{F}_r, z_j \in A_7, |K_j| \in \{7, 5\}), \quad (7)$$

so every A_7 -module over a field of characteristic r on which all elements of orders 7 and 5 are fixed-point-free is zero.

Proof. The characteristics 5 and 7 are excluded by Lemma 2.3, and $r > 7$ by Lemma 3.1. For $r \in \{2, 3\}$ and a module V with $V^K = 0$ for all subgroups K of order 7 or 5, (7) gives $v = \sum_j c_j z_j s_{K_j} v = 0$, as $s_{K_j} v \in V^{K_j} = 0$. This uses neither simplicity, nor a change of field, nor the converse half of Lemma 6.1. The identities (7) are those of the certificate files described in (2) below. $\square$

Three independent table-free certificates support Proposition 6.2. (1) *Rank*. `ideal_rank.py` computes $\dim_{\mathbb{F}_r} I$ by exact elimination modulo r on the 106,704 coset vectors of the 246 conjugates of P and H (120 and 126), in a seeded random order, and finds $\dim I = 2520 = |A_7|$ for $r = 2$ and $r = 3$, so $1 \in I$. For the ideal of P alone it finds codimension $36 = 6^2$ in both characteristics, for H alone $32 = 4^2 + 4^2$ ($r = 2$) and 0 ($r = 3$), and for $p = 5$, $r = 2$ codimension $8 = 2^2 + 2^2$, matching the two 2-dimensional modules of Proposition 5.1. (2) *Explicit certificates*. The first verification run recorded coefficients for (7) in the files `OWR-14299292-011_A_cert_p7_r2.json` (1251 terms: 525 with K_j generated by a 7-cycle and 726 with K_j generated by a 5-cycle; 240 distinct subgroups) and `..._r3.json` (1675 terms: 706 and 969; 245 distinct subgroups). Each term gives a generator of K_j , the representative z_j and c_j ; permutations of $\{0, \dots, 6\}$ compose as $(x \circ y)(i) = x(y(i))$, and $z_j K_j = \{z_j \circ k\}$ (recorded in each file). Part B of `check_note.py` checks the cycle types and signs and expands the sum in $\mathbb{F}_r A_7$: it is the identity. (3) *Simple modules*. The second verification run computed the simple $\mathbb{F}_r A_7$ -modules with its own MeatAxe-type program (Norton's irreducibility test [Par84, HR94], applied to permutation modules on points, 2- and 3-subsets and pairs (2-subset, point)); the list is complete because the dimensions of the endomorphism rings add up to the number of r -regular classes. With $[\dim V^g, \dim V^h]$:

r	simple $\mathbb{F}_r A_7$ -modules: dimension $[\dim V^g, \dim V^h]$	both fixed-point-free
2	1 [1, 1], 4 [1, 0], 4' [1, 0], 6 [0, 2], 14 [2, 2], 20 [2, 4]	none
3	1 [1, 1], 6 [0, 2], 13 [1, 1], 15 [3, 3], 20 [2, 4] (endomorphisms $\mathbb{F}_9$; 10 + 10' over $\mathbb{F}_9$)	none

With Lemma 2.1 this decides $p = 7$ once more. By these lists no simple module of A_7 in characteristic 2 or 3 has dimension divisible by $12 = \text{lcm}(p - 3, \frac{p-1}{2})$, so Lemma 2.4(ii) alone gives the answer; the same holds for S_7 and $(p - 1)(p - 3)/2 = 12$ by the published tables of S_7 (`decomp_check.py` 7 2 and 7 3), which agree with the lists and are supplementary only.

7 The cases $p = 13$ and $p = 19$

Proposition 7.1. (a) *No representation as in the question exists for $p = 13$ in characteristic 7, for $p = 19$ in characteristics 11 and 13, and for $p \in \{13, 19\}$ in characteristics p , $p - 2$ and $r > p$.*

(b) *Conditional statement. Assume that the following decomposition matrices published by the Modular Atlas project [Mod] are correct: those of S_{13} in characteristics 2, 3, 5, of S_{19} in characteristic 2, and of S_{17} in characteristics 3, 5, 7. Then no representation as in the question exists for $p = 13$ in characteristics 2, 3, 5, and for $p = 19$ in characteristics 2, 3, 5, 7.*

Proof. (a) is Lemmas 2.3, 3.1 and 3.2 ($7 > 13/2$ and $19/2 < 11 < 13 < 17$). (b) Let M be the r -modular decomposition matrix of S_n (rows: all partitions of n ; columns: the irreducible Brauer characters) and X the table of ordinary characters on the r -regular classes (Murnaghan–Nakayama). The Brauer character table B is the solution of $MB = X$, and (1) gives the fixed-space dimensions. The programs find:

- $p = 13$, $r \in \{2, 3, 5\}$ (`A13.2partmod2`, 3, 5; 101 rows, 18, 44, 76 columns): the 13-cycle is fixed-point-free exactly on $N = D^{(12,1)}$ and, for odd r , on N' ; the 11-cycle h on no simple FS_{13} -module.
- $p = 19$, $r = 2$ (`A19.2partmod2`; 490×54): the 19-cycle is fixed-point-free exactly on $D^{(18,1)}$, the 17-cycle h on no simple FS_{19} -module.
- $p = 19$, $r \in \{3, 5, 7\}$ (`A17.2partmod3`, 5, 7): exactly two simple FS_{17} -modules have a 17-cycle fixed-point-free, and their Brauer characters equal $\pi - 1$ and $\text{sgn} \cdot (\pi - 1)$ on all r -regular classes,

so they are N and N' ; Lemma 4.1 ($q = 17$) shows that no nonzero FS_{19} -module has h fixed-point-free. The linear system of `twist_test.py` (70, 127, 153 equations in 4 unknowns) has only the zero solution.

Lemma 2.4(iii) completes the proof. $\square$

Cross-checks (files not used in the proof). For $p = 13$ and $r \in \{3, 5\}$, Lemma 4.1 with the tables of S_{11} gives the same conclusion; the 11-cycle-fixed-point-free simple FS_{11} -modules are N and N' (also for $r = 7$). For $p = 19$ and $r \in \{5, 7\}$ the tables of S_{18} show that no simple FS_{18} -module has a 17-cycle fixed-point-free; for $r = 3$ there are two ($D^{(17,1)}$ and $D^{(9,8,1)}$, of dimension 16). For $S_{13} \bmod 7$ and $S_{17} \bmod 11, 13$ (where $r > n/2$) the largest $|\varphi|$ at an n -cycle is 1, within the bound 2 of the proof of Lemma 3.2; for $S_{13} \bmod 2, 3, 5$ it is 2, and for $S_{19} \bmod 2$ it is 4, where that bound does not apply.

What the data steps check, and what they do not. The programs check that the parse is complete, with rows labelled by all partitions in reverse lexicographic order; James's unitriangularity [Jam78] (each column starts with a 1 in a row labelled by an r -regular partition, these are all the r -regular partitions, and their rows form a lower unitriangular block; checked by `brauer_lib.py`, used by `identify_fpf.py` and `twist_test.py`, while `decomp_check.py` checks full column rank); that $MB = X$ holds over all partitions and all r -regular classes; and that B is integral with positive degrees. All three verification runs repeated this with their own parsers and solvers, with the same results; in the second, one altered entry of the matrix of $S_{13} \bmod 3$ produced 41 inconsistent pairs (row, class). These checks do not see every error: a changed entry in the row of an r -core (a block of defect zero, whose column has no other nonzero entry) leaves $MB = X$ consistent. The third verification run (Section 9) therefore also checked that nonzero decomposition numbers occur only between partitions with the same r -core (no violation in the 18 tables), and increased each entry of the seven tables used in the proof by one, in turn. Without this block check, 233 of the 7,676 changes for $S_{13} \bmod 5$ and 172, 760 and 4,830 of those for $S_{17} \bmod 3, 5$ and 7 go unnoticed (none for $S_{13} \bmod 2, 3$ and $S_{19} \bmod 2$); with it, every one of them is detected. No check of this kind detects a consistent but wrong table, so the checks do not prove the decomposition numbers; hence the conditional form of Proposition 7.1(b) and of the answers for $p = 13$ and 19.

Provenance. The files were downloaded on 21 October 2026 from the Modular Atlas database [Mod]; its index page states that it was created automatically from the character table library of GAP [Bre], whose home page lists version 1.3.11 on that date. Which version produced the files is not stated (the directory page for A_{13} was created by GAP on 1 March 2023). The SHA-256 of all files are given in Appendix A; the second verification run downloaded all 18 files again and found the same values. Primary references for the original computations of these tables were not traced: the references page of the Modular Atlas reproduces those of the Atlas of Brauer Characters, which covers the simple groups of order less than 10^9 and so not A_{13} , A_{17} , A_{19} , and the library's information page for A_{13} names no source for its Brauer tables.

8 The open cases

By Corollary 3.3 the open cases (Problem 1.2) are the twin primes $p \geq 31$ with a prime $r < p/2$, where a simple FS_p -module as required would have dimension divisible by $(p-1)(p-3)/2$ (420 for $p = 31$). In every computed case with $p \geq 7$ the answer is no. For $p = 13$ and $p = 19$ and every prime $r < p$, no simple FS_p -module even has h fixed-point-free according to the tables (for $r = p-2$ by Lemma 2.3); for $p = 7$ a simple module on which a 5-cycle is fixed-point-free exists only in characteristic 2, namely $D^{(4,3)}$, of dimension 8, which restricts to $4 + 4'$ on A_7 , and the 7-cycle has fixed vectors on it. This is

not a proof. For odd r , Lemma 4.1 reduces the question to a case of Zalesski's Problem 4 (Remark 4.2); for $r = 2$ the method of Lemma 4.1 does not apply, and no reduction for $r = 2$ is known to us.

Remark 8.1 (a conditional answer). Zalesski expects [Zal25b, p. 6] that the representations of Problem 4 there have dimension less than the degree. If this holds for A_p , the answer to Lee's question is no for every $p \geq 7$: if g is fixed-point-free on a simple FA_p -module W , the minimal polynomial of g has degree less than p , so $\dim W < p$; but if h is also fixed-point-free, $\dim W$ is a positive multiple of $\text{lcm}(p-3, \frac{p-1}{2})$ (Lemma 2.4(ii)), which is 12 for $p = 7$ and at least $(p-1)(p-3)/4 > p$ for $p \geq 13$. This is conditional on an expectation that is not proved; it was suggested by the second verification run.

Conclusion. The note gives a *partial answer*: yes for $p = 5$, no for $p = 7, 13, 19$ (13 and 19 conditional on published decomposition matrices); open for $p \geq 31$. The negative answers for $p = 13$ with $r \in \{2, 3, 5\}$ and for $p = 19$ with $r \in \{2, 3, 5, 7\}$ rest on the published decomposition matrices; $p = 5$, $p = 7$, all $r \geq p/2$, $p = 13$ with $r = 7$ and $p = 19$ with $r \in \{11, 13\}$ are table-free.

9 Computations and reproducibility

The proofs of the lemmas and of Proposition 5.1 use no computation; Proposition 6.2 rests on the identities (7), and Proposition 7.1(b) on computations with the published tables. The package contains: *the exact program of this note*, `check_note.py` (Python 3, standard library only, about 13 s), which checks (A) the isomorphism and the fixed-point-free action of Proposition 5.1, (B) both certificate files of Proposition 6.2, with a negative control, (C) the class and coset counts for A_7 , (D) the arithmetic of Lemma 2.4 and Zolotarev's lemma for the 15 twin primes $p \leq 200$, (E) the values (2) for all partitions of 5, 7, 13, 19 and the case $p = 5$ of Lemma 3.1, (F) the hook count of Lemma 3.2 for all 124 pairs (p, r) with $p \leq 200$ and $p/2 < r < p-2$, with a control showing that it fails for $r < p/2$, and (G) the constants of Lemma 4.1 and the conjugacy of yt and z for $q = 11, 17$; all checks pass. The programs with which the results were first obtained (`original/`; Python with numpy, sympy and pypdf): `p5_sl24_module.py`, `ideal_rank.py` (seconds), `decomp_check.py`, `brauer_fpf.py`, `brauer_lib.py` and `twist_test.py` (under a second per table), with their recorded outputs, which were reproduced. `identify_fpf.py`, written with this note, identifies the fixed-point-free modules by their Brauer characters for all 18 tables. The tables are not redistributed; `tables/fetch_tables.py` downloads them, checks the SHA-256 of Appendix A and writes the text extractions that the programs read. The certificate files and the MeatAxe-type program of the second verification run are included with their outputs, and the programs of the third run, with their outputs, in `independent_run_2/`.

Verification. The results were first obtained, with proofs and with the programs in `original/`. Three independent verification runs, all AI-assisted, followed; each re-derived the proofs and wrote its own programs. The first run re-derived all lemmas and the case analysis, re-ran the program for $p = 5$, computed the certificates of Proposition 6.2 with its own program, reproduced every number of Section 7 with its own parser and solver, compared the Brauer characters of the fixed-point-free modules with $\pi - 1$ and $\text{sgn} \cdot (\pi - 1)$, and read Problem 4 and the reference to Thompson in [Zal25b]. It asked for the integrality argument in Lemma 2.4(i), for the conditional to be stated in the headline, for the identification by Brauer characters, and for the remark on the size of the core in Lemma 3.2. The second run computed, before reading the original programs, the simple modules of A_5 and A_7 with its own MeatAxe-type program, all ideal dimensions with all rows, and the isomorphism of Proposition 5.1 with a negative control; it re-ran all original programs and reproduced their outputs; it downloaded the 18 tables again (same SHA-256) and processed them with its own parser and solver, with the negative control of Section 7; and it checked the hook count, the values (2), the constants of Lemma 4.1 and the bibliographic data. It asked for the attribution of the case $p/2 < r < p-2$,

for the provenance of the tables and for a clarification in Section 8, and suggested Remark 8.1. Both runs confirmed the results within the stated scope and found no mathematical error. All corrections they required were made.

The third run examined the final text of this note line by line before any change. It read again the problem session and the first page of [DENRD25], the corpus record, pp. 4–6 and the reference list of [Zal25b] and the pages of [Mod, Bre] cited here, re-derived every proof, and checked the bibliographic data with Crossref. Its own programs, written from the text, check both certificate files of Proposition 6.2 with negative controls; process all 18 tables with their own text extraction and parser, identify the fixed-point-free modules by their Brauer characters and add the block check of Section 7; test every single changed entry of the seven tables used in the proof; recompute the finite statements of Lemmas 2.4, 3.2 and 4.1 and (2); and check Proposition 5.1 directly in $SL_2(4)$. It downloaded three of the tables again (same SHA-256), ran the fetch-and-verify program on local copies with a negative control, re-ran the whole package from the archive (all 48 recorded outputs were reproduced) and repeated part of the literature search. It confirmed the results within the stated scope, found no mathematical error and required seven corrections, which were made: neutral wording for the proposer, the date of the problem session, the counts of the certificate files, the account of what the table checks detect, this description, the account of the searches and sources, and the verification report.

Scope and priority. The note gives a partial answer to Lee’s question [DENRD25, Problem 1]: yes for $p = 5$, no for $p = 7, 13, 19$ (13 and 19 conditional on published decomposition matrices); open for $p \geq 31$. For every twin prime $p \geq 7$ all characteristics $r \geq p/2$ are excluded without tables, and for odd r the remaining cases are reduced to the classification of Remark 4.2. The question and the remark on $r > p$ are due to Lee; Lemma 3.1 is that remark with its proof. Lemma 3.2 is a standard Brauer-tree argument; for a p -cycle alone the case $\ell > n/2$ is attributed by Zalesski [Zal25b, p. 6] to Thompson [Tho08, Lemma 3.2], which we did not read (the publisher’s site refused automated access with HTTP status 403, also to the third verification run, and this was not worked around). The other lemmas use standard tools: Brauer characters and Clifford theory [Nav98], blocks of defect one and Brauer trees [Bra42, Dad66, Fei82], the symmetric groups [Jam78], and the MeatAxe [Par84, HR94]. Lemma 4.1 was not found in print in this form, but we claim no priority for it. What was read: the problem session of [DENRD25] in the open publisher file; pp. 4–6 of [Zal25b] and its reference to [Tho08]; the index, directory and references pages of [Mod] and two pages of [Bre]; the full texts of [CLM⁺25, FL26] were searched for alternating groups by the second verification run; of [Zal25a, Vel24] and the other works found only the abstracts were read. The textbook results cited from [Nav98, Bra42, Dad66, Fei82, Jam78, Par84, HR94] were not re-read for this note. The searches (21 October 2026, repeated in part by the third verification run): the arXiv API (alternating groups and the eigenvalue 1, fixed-point-free representations, twin primes, papers of Zalesski and of Lee, unisingular groups, Brauer characters at cycles), zbMATH, Semantic Scholar (including the 5 works citing [Zal25b]; one keyword search of the second run was refused with HTTP status 429), Crossref and OpenAlex (no work citing [DENRD25]), and four web searches. No printed answer to the question was found in these sources; a search that finds nothing is not a proof of novelty.

AI-assisted tools were used to search the problem corpus and the literature, to find the arguments, to write and run the verification programs, and to help prepare this manuscript. The author has reviewed the proofs, the computations and the sources and is responsible for all claims. The note has not been peer reviewed.

A The data files

Downloaded on 21 October 2026 from [Mod]: the file `tex/A<n>/A<n>.2partmod<r>.pdf` is the r -modular decomposition matrix of S_n , rows labelled by partitions. “Proof” marks the seven files on which Proposition 7.1(b) rests. The last row is the publisher file of [DENRD25] (<https://ems.press/content/serial-article-files/51860>).

file	matrix	role	SHA-256 of the PDF file
A7.2partmod2	$S_7, 15 \times 5$	check	a50809b8d2ecb9573a327f4558db7d747996ebfab67317488c0cdbbed6426d8f
A7.2partmod3	$S_7, 15 \times 9$	check	524faffa36f864877d54f2a3cb8aacbdb26e438b061cf8f49f9a7ca2d14c96e0
A11.2partmod3	$S_{11}, 56 \times 27$	check	d6ae9b7e61f8992293366503979c74e78e50da0cc2a81bb024c77bcb518db9e5
A11.2partmod5	$S_{11}, 56 \times 44$	check	6f17ca3b15279173644b9b098898151d1faae9edfe8b6da5113926b91f6ebfde
A11.2partmod7	$S_{11}, 56 \times 51$	check	9267033d4ed3e309d123bfbf65f31f9f05178c2c0772b87b8631142d99fb3d48
A13.2partmod2	$S_{13}, 101 \times 18$	proof	58a4e454f90d927cb93483271b7019e289d5ea2eff44a84ff4ed2d2ad9d7a7db
A13.2partmod3	$S_{13}, 101 \times 44$	proof	9599b3526ee1d048418c46304f26dad13937841ed38912777ba6a5b4b081244d
A13.2partmod5	$S_{13}, 101 \times 76$	proof	8121dc6776bc1cf857511f7b21dd789ac1b568395d7fd389e2b271cd857fd96f
A13.2partmod7	$S_{13}, 101 \times 90$	check	96effe4c1c0e37bebba2daa2f1f001f02b539000f89dcb3808aad548b426e6bd
A17.2partmod3	$S_{17}, 297 \times 108$	proof	49454d228c20ba063cd87a10bfbe79914f74492fd3c8ed21672bf0987ee31105
A17.2partmod5	$S_{17}, 297 \times 205$	proof	8ee7c34a5e1ca52e8dc691eed426942462009cbcb1afc66f1da816efe19915862
A17.2partmod7	$S_{17}, 297 \times 252$	proof	49d0c35916406778aef0d5056a55c7b176e4dde9bb8dca39f1b7cedf13feb808
A17.2partmod11	$S_{17}, 297 \times 286$	check	5fc25225111a24ebb566ef89f6798443b3a5353a33a5dd6b7535708edd44aca8
A17.2partmod13	$S_{17}, 297 \times 292$	check	6dfeba57f27e20feef40c053598eccd4a9ea7ee2af551bae6a90d20b5671dae9
A18.2partmod3	$S_{18}, 385 \times 135$	check	8fc588972ae2e90d384c563e8119e7d35f519f9405a3f5ce93cf75a5d355cdb3
A18.2partmod5	$S_{18}, 385 \times 262$	check	ac3c9926ea372307aad039fb7c223c8b93c5e0adb6ad62eab8511c4622bc8e80
A18.2partmod7	$S_{18}, 385 \times 324$	check	b8cb41d9821925b89adca1c98e096ae8e9ab925862d9f05f37150ebe59239552
A19.2partmod2	$S_{19}, 490 \times 54$	proof	32f5bbfa517eb8ef7bc66c4e907e1f776a17735d7a5ff79ff411c485c7f9f730
OWR 27/2025	source, 62 pages	source	818c47a49ce2fd05b13dd30e2bd9826f869fedf5e8b1c3a9d6d77bf0c1e356b0

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