

Hypercyclic Subspaces on ω and a Partial Answer to a Question of Petersson

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Abstract

In the report of the 2006 Oberwolfach mini-workshop on hypercyclicity, H. Petersson asked two questions about a (hereditarily) hypercyclic operator T on a separable Fréchet space X . First, does $\text{HC}(T) \cup \{0\}$ contain the range of an injective operator $S \in L(X)$? Second, for such an S , is every linearly independent n -tuple in $\text{Im } S$ hypercyclic for $T \oplus \cdots \oplus T$? We give a partial answer. Our main result concerns the space $\omega = \mathbb{K}^{\mathbb{N}}$. For every hypercyclic operator T on ω there is an injective $S \in L(\omega)$ such that $(Sx_1, \dots, Sx_n)$ is hypercyclic for the n -fold direct sum of T whenever $x_1, \dots, x_n$ are linearly independent. Its range is closed, so every hypercyclic operator on ω has a hypercyclic subspace. The proof uses a special case of a lemma of Shkarin (2011, Lemma 1.5). The hypercyclic-subspace consequence also follows quickly from that lemma combined with a criterion of Menet (2013, Theorem 4.8), and it answers, for an operator on ω and its iterates, a question raised by Menet. On separable Banach spaces, a folklore argument with the entire functional calculus answers the first question positively for every hypercyclic operator. For weakly mixing operators on spaces with a continuous norm, the source itself answers it. In its universal reading, for every such S , the second question has a negative answer: in all these settings there is an S as in the first question whose range contains a pair (x, Tx) , and such a pair is never hypercyclic for $T \oplus T$. In its existential reading, for some such S , it has a positive answer exactly for the weakly mixing operators, on ω and on spaces with a continuous norm. We leave the first question open for hypercyclic operators that are not weakly mixing on non-normable Fréchet spaces with a continuous norm, and for Fréchet spaces without a continuous norm other than ω . This is an unrefereed note.

1 Introduction

Let X be a separable Fréchet space over $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$ and let $L(X)$ be the algebra of continuous linear operators on X . An operator $T \in L(X)$ is *hypercyclic* if some $x \in X$ has a dense orbit $\{T^k x : k \geq 0\}$; the set of such vectors is $\text{HC}(T)$. For $n \geq 1$ we write $T^{\oplus n} = T \oplus \cdots \oplus T$ for the n -fold direct sum acting on X^n . T is *weakly mixing* if $T^{\oplus 2}$ is hypercyclic. A *hypercyclic subspace* for T is a closed infinite-dimensional subspace M with $M \setminus \{0\} \subseteq \text{HC}(T)$.

The question. In his abstract for the Oberwolfach mini-workshop *Hypercyclicity and Linear Chaos* [Pet06], Petersson recalls that, by Bès and Peris [BP99], T satisfies the Hypercyclicity Criterion if and only if T is hereditarily hypercyclic, if and only if T is weakly mixing. He states, without a reference, that when X has a continuous norm these conditions are also equivalent to the following:

- (v) $L_T : R \mapsto TR$ is hypercyclic on $L(X)$ for the strong operator topology (SOT);

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- (vi) there is $S \in L(X)$ such that $(Sx_1, \dots, Sx_n)$ is hypercyclic for $T^{\oplus n}$ for every linearly independent set $\{x_1, \dots, x_n\} \subseteq X$.

He also notes that S is an SOT-hypercyclic vector for L_T if and only if S has the property in (vi), with or without a continuous norm (Lemma 2.5 below). So $\text{Im } S$ is an infinite-dimensional subspace all of whose linearly independent n -tuples are hypercyclic for $T^{\oplus n}$. He then asks [Pet06, p. 2260] whether, for any (hereditarily) hypercyclic T , the set $\text{HC}(T)$ contains “the range $\text{Im } S$ of a one-to-one operator” $S \in L(X)$. He also asks whether, for such an S , every linearly independent n -tuple in $\text{Im } S$ is hypercyclic for $T^{\oplus n}$. The problem is record OWR-1323-008 of the UnsolvedMath dataset [ula26], where it is stated for hereditarily hypercyclic T . Hypercyclicity of L_T for the SOT was studied by Chan [Cha99] on Hilbert spaces, by Chan and Taylor [CT01] on Banach spaces, and with tensor-product methods by Martínez-Giménez and Peris [MGP03] and by Bonet, Martínez-Giménez and Peris [BMGP04]. On separable Fréchet spaces with a continuous norm, the implication from weak mixing to (v) follows from [BMGP04] together with [BP99] (Remark 4.4).

Readings. Since $0 \in \text{Im } S$ and $0 \notin \text{HC}(T)$, the first clause must be read as $\text{Im } S \setminus \{0\} \subseteq \text{HC}(T)$. In the second clause the source writes “if T is such a mapping”. We read this as referring to S . It can also be read as referring to an operator T for which the first clause holds, with S the operator from the first clause. Both readings lead to the questions below. We call an injective $S \in L(X)$ with $\text{Im } S \setminus \{0\} \subseteq \text{HC}(T)$ *admissible for T* , and we say that S has *property (vi) for T* if it satisfies the condition in (vi). We consider three questions.

(Q1) Is there an admissible operator for T ?

(Q2) Universal reading: is every linearly independent n -tuple in $\text{Im } S$ hypercyclic for $T^{\oplus n}$, for every admissible S ?

(Q2') Existential reading: is there an injective $S \in L(X)$ such that every linearly independent n -tuple in $\text{Im } S$ is hypercyclic for $T^{\oplus n}$? For injective S , this is property (vi).

The source itself answers (Q1) and (Q2') positively for weakly mixing T when X has a continuous norm: take S as in (vi). What the source leaves open is (Q2), and (Q1) for operators that are not weakly mixing or for spaces without a continuous norm.

Results. Let $\omega = \mathbb{K}^{\mathbb{N}}$ with the product topology. It is the basic example of a Fréchet space without a continuous norm. Its dual is the space $\varphi = \mathbb{K}^{(\mathbb{N})}$ of finitely supported sequences, and every $T \in L(\omega)$ has a transpose $T': \varphi \rightarrow \varphi$ (Section 4).

Theorem 1.1. *For $T \in L(\omega)$ the following are equivalent.*

- (a) T is hypercyclic.
- (b) $p(T')$ is injective for every nonzero polynomial $p \in \mathbb{K}[t]$.
- (c) There is an injective $S \in L(\omega)$ with property (vi) for T .

In this case $\text{Im } S$ is closed and infinite-dimensional. So $\text{Im } S$ is a hypercyclic subspace for T , every linearly independent n -tuple in $\text{Im } S$ is hypercyclic for $T^{\oplus n}$, and S is a hypercyclic vector for L_T on $(L(\omega), \text{SOT})$.

The equivalence (a) $\Leftrightarrow$ (b) is known. Herzog and Lemmert [HL93] proved it for $\mathbb{K} = \mathbb{C}$ in the form $\sigma_p(T') = \emptyset$. According to its zbMATH review, their paper states the criterion as torsion-freeness of the dual of ω as a $\mathbb{C}[\lambda]$ -module under the transpose, which is condition (b). Shkarin proved it for $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$ [Shk15, Theorems 1.7 and 1.8], [Shk11, Corollary 1.2], together with the fact that every hypercyclic operator on ω is mixing. By [Shk15, Corollary 1.10], ω is, up to isomorphism, the only separable infinite-dimensional Fréchet space that supports no hypercyclic non-mixing operator. Bayart and Matheron [BM07] had shown that every hypercyclic operator on ω satisfies the Hypercyclicity Criterion. The implication (a) $\Rightarrow$ (c) is the contribution of this note. We have not found it in the literature. In particular, *every hypercyclic operator on ω has a hypercyclic subspace*. This consequence, but not property (vi), also follows quickly from Shkarin's Lemma 1.5 in [Shk11] combined with a criterion of Menet [Men13, Theorem 4.8]. Remark 4.2 explains this, and why it answers Menet's question in [Men13, Remark 4.10] for an operator on ω and its iterates.

Earlier results give hypercyclic subspaces on ω for particular classes. Bès and Conejero [BC06] treated every non-constant polynomial of a backward weighted shift, as described in [Men13], and showed that countably many non-constant polynomials of a fixed backward shift have a common hypercyclic subspace, as described in [Pet07, BS24]. Salas [Sal11] proved that upper staircase matrices are hypercyclic, and chaotic when $\mathbb{K} = \mathbb{C}$, and that countably many strictly upper triangular upper staircase matrices have a common hypercyclic subspace. We know this from its zbMATH review and, for the first part, from [BS24]. Menet's Corollary 4.12 [Men13] gives a large class, which includes $P(B_w) + \sum_k \alpha_k S_w^k$. Petersson [Pet07, Theorem 2] proved that if a sequence of operators on ω has a hypercyclic subspace H , then some injective $S \in L(\omega)$ has $\text{Im } S \subseteq H$. For a single operator, Theorem 1.1 shows that such an H always exists, and it gives S with the stronger property (vi); see Remark 4.3.

On Banach spaces the first clause has a folklore answer.

Theorem 1.2. *Let X be a separable infinite-dimensional Banach space, let $T \in L(X)$ be hypercyclic, and let $x_0 \in \text{HC}(T)$. Let $(e_k, \varphi_k)_{k \geq 0}$ be a Markushevich basis of X with $\|\varphi_k\| = 1$, and put*

$$Su = \sum_{k \geq 0} \frac{\varphi_k(u)}{k!} T^k x_0 \quad (u \in X).$$

Then S is a nuclear operator, injective, with dense range, and $\text{Im } S \setminus \{0\} \subseteq \text{HC}(T)$. Moreover $T^j x_0 = j! S e_j \in \text{Im } S$ for every j , and for every $n \geq 2$ the linearly independent n -tuple $(x_0, T x_0, \dots, T^{n-1} x_0)$ is not hypercyclic for $T^{\oplus n}$.

Theorem 1.3. *Let X be a Fréchet space and $T \in L(X)$. Suppose $S_0 \in L(X)$ satisfies the condition in (vi) for $n \leq 2$. Let $\psi \in X'$ and $e \in X$ with $\psi(e) = 1$, let $a \in \ker \psi \setminus \{0\}$, and put*

$$S'u = S_0(u - \psi(u)e) + \psi(u) T S_0 a \quad (u \in X),$$

that is, $S' = S_0 + (T S_0 a - S_0 e) \otimes \psi$. Then S' is admissible for T . Its range contains $x = S_0 a = S' a$ and $T x = S' e$, and the linearly independent pair $(x, T x)$ is not hypercyclic for $T \oplus T$.

Corollary 1.4. *The questions (Q1), (Q2) and (Q2') have the following answers.*

(i) (Q1) has a positive answer for every hypercyclic operator on a separable Banach space (Theorem 1.2), for every hypercyclic operator on ω (Theorem 1.1), and, by the source, for every weakly mixing operator on a Fréchet space with a continuous norm.

(ii) (Q2) has a negative answer for every operator T in (i).

- (iii) If X has a continuous norm or $X = \omega$, then $(Q2')$ has a positive answer for T if and only if T is weakly mixing. On ω this holds for every hypercyclic T . On a separable Banach space it holds if and only if T satisfies the Hypercyclicity Criterion.

Hypercyclic operators that are not weakly mixing exist. De La Rosa and Read [DR09] gave the first example, on a specially constructed Banach space, and Bayart and Matheron [BM07] gave examples on ℓ^p ($1 \leq p < \infty$), c_0 and separable Hilbert space. For such operators (Q1) holds by Theorem 1.2, while (Q2) and (Q2') fail for every S . The refutation of (Q2) is elementary, and Theorem 1.2 is folklore. The substance of the note is Theorem 1.1. The cases of (Q1) that we leave open are listed in Section 6.

2 Preliminaries

All topological vector spaces are Hausdorff. The first lemma is classical (Rolewicz [Rol69]; see also [GEP11, Chapter 2]). We include the short proof.

Lemma 2.1. *For $m \geq 1$, no linear map $B: \mathbb{K}^m \rightarrow \mathbb{K}^m$ is hypercyclic.*

Proof. Suppose $\{B^k y : k \geq 0\}$ is dense. Assume first that $B^\top$ has an eigenvalue $\lambda \in \mathbb{K}$, with eigenvector $\mu \neq 0$. The map $z \mapsto \mu^\top z$ is a continuous surjection $\mathbb{K}^m \rightarrow \mathbb{K}$, so $\{\mu^\top B^k y\} = \{\lambda^k c\}$ with $c = \mu^\top y$ is dense in $\mathbb{K}$. This is impossible. If $|\lambda| \leq 1$ or $c = 0$ the set is bounded. If $|\lambda| > 1$ and $c \neq 0$ it meets the unit disc in finitely many points. Otherwise $\mathbb{K} = \mathbb{R}$ and $B^\top$ has a non-real eigenvalue λ with eigenvector $\mu_1 + i\mu_2$, where $\mu_1, \mu_2 \in \mathbb{R}^m$ are linearly independent. Then $z \mapsto (\mu_1 + i\mu_2)^\top z$ is a continuous $\mathbb{R}$ -linear surjection $\mathbb{R}^m \rightarrow \mathbb{C}$. It maps the orbit onto $\{\lambda^k c\}$, which again cannot be dense in $\mathbb{C}$. $\square$

Lemma 2.2 (Bourdon, Bès, Wengenroth). *Let X be a locally convex space and $T \in L(X)$ hypercyclic. Then $p(T)$ has dense range for every nonzero polynomial $p \in \mathbb{K}[t]$.*

Proof. This is proved in [Bou93] (complex Banach spaces), [Bè99] (real scalars) and [Wen03] (general spaces). For completeness: suppose $p(T)X$ is not dense. By the Hahn–Banach theorem there is $0 \neq \chi \in X'$ with $\chi \circ p(T) = 0$, so $d = \deg p \geq 1$. Then $F = \text{span}\{\chi, \chi \circ T, \dots, \chi \circ T^{d-1}\}$ is a nonzero finite-dimensional subspace of X' with $\chi' \circ T \in F$ for all $\chi' \in F$. Let $\chi_1, \dots, \chi_m$ be a basis of F and write $\chi_i \circ T = \sum_j \beta_{ij} \chi_j$. The map $J = (\chi_1, \dots, \chi_m): X \rightarrow \mathbb{K}^m$ is continuous and surjective, and $JT = BJ$ with $B = (\beta_{ij})$. So J maps a dense orbit of T onto a dense orbit of B , contradicting Lemma 2.1. $\square$

Lemma 2.3. *Let Y, X be topological vector spaces, $R \in L(Y)$, $T \in L(X)$, and let $P: Y \rightarrow X$ be continuous with dense range and $PR = TP$. Then $P(\text{HC}(R)) \subseteq \text{HC}(T)$. In particular, if $F \in L(X)$ commutes with T and has dense range, then $F(\text{HC}(T)) \subseteq \text{HC}(T)$. If X is locally convex, then $p(T)(\text{HC}(T)) \subseteq \text{HC}(T)$ for every nonzero polynomial p .*

Proof. If $y \in \text{HC}(R)$, the orbit of Py under T is P of the orbit of y . By continuity its closure contains $P(Y)$, which is dense. The last statement follows from Lemma 2.2. $\square$

Lemma 2.4. *Let $X \neq \{0\}$ be a topological vector space, $T \in L(X)$, $x \in X$ and $n \geq 2$. Then $(x, Tx, \dots, T^{n-1}x)$ is not hypercyclic for $T^{\oplus n}$. If $x \in \text{HC}(T)$ and X is locally convex, these n vectors are linearly independent.*

Proof. The orbit lies in $G_n = \{(y, Ty, \dots, T^{n-1}y) : y \in X\}$. G_n is closed because T is continuous and X is Hausdorff. It is proper, since $(0, z, 0, \dots, 0) \notin G_n$ for $z \neq 0$. If $\sum_{j < n} c_j T^j x = 0$ with $p(t) = \sum_j c_j t^j \neq 0$, then $p(T)x = 0$, but $p(T)x \in \text{HC}(T)$ by Lemma 2.3. $\square$

The next lemma is the remark of Petersson recalled in the introduction.

Lemma 2.5. *Let X be a locally convex space and $S, T \in L(X)$. Then $\{T^k S : k \geq 0\}$ is SOT-dense in $L(X)$ if and only if S has property (vi) for T .*

Proof. Suppose $\{T^k S\}$ is SOT-dense. Let $x_1, \dots, x_n$ be linearly independent, $y_1, \dots, y_n \in X$, and let U be a 0-neighbourhood. By the Hahn–Banach theorem there are $\chi_i \in X'$ with $\chi_i(x_j) = \delta_{ij}$. Put $R = \sum_i \chi_i(\cdot) y_i \in L(X)$, so $Rx_j = y_j$. SOT-density gives k with $T^k Sx_j - y_j \in U$ for all j . Conversely, assume (vi). Let $R \in L(X)$, $z_1, \dots, z_r \in X$ not all zero, and let U be an absolutely convex 0-neighbourhood. Choose a basis $x_1, \dots, x_n$ of $\text{span}\{z_j\}$ and write $z_j = \sum_l \alpha_{jl} x_l$. Let $c = 1 + \max_j \sum_l |\alpha_{jl}|$. Since $(Sx_l)_l$ is hypercyclic for $T^{\oplus n}$, there is k with $T^k Sx_l - Rx_l \in c^{-1}U$ for all l . Then $T^k S z_j - R z_j = \sum_l \alpha_{jl} (T^k Sx_l - Rx_l) \in U$ for all j . $\square$

3 Banach spaces

The next lemma is the entire functional calculus behind the classical fact that $\{f(T)x_0 : f \text{ entire}\} \setminus \{0\} \subseteq \text{HC}(T)$. For this fact see [Her91, Bou93, Bè99] and [ABGPSS16, Section 4.3], as quoted in [CRV26].

Lemma 3.1. *Let X be a Banach space, $T \in L(X)$ hypercyclic, and let $f(z) = \sum_k c_k z^k$ be a nonzero entire function with $c_k \in \mathbb{K}$. Then $f(T) = \sum_k c_k T^k$ converges absolutely in norm, commutes with T and has dense range. Hence $f(T)(\text{HC}(T)) \subseteq \text{HC}(T)$.*

Proof. Since the zeros of f are isolated, there is $r > \|T\|$ such that f has no zero on $|z| = r$. Let p be the monic polynomial whose roots are the zeros of f in $|z| < r$, with multiplicity. If $\mathbb{K} = \mathbb{R}$, the non-real zeros of f come in conjugate pairs of equal multiplicity, so p has real coefficients. Then $g = f/p$ is entire, with Taylor coefficients in $\mathbb{K}$, and it has no zero in $|z| \leq r$. So $1/g$ is holomorphic on a disc $|z| < r'$ with $r' > r$. Its Taylor series $\sum_k b_k z^k$, again with coefficients in $\mathbb{K}$, converges absolutely at $\|T\|$. All series $\sum c_k T^k$, $g(T)$ and $h(T) = \sum b_k T^k$ converge absolutely in norm. Cauchy products of absolutely convergent series in the Banach algebra $L(X)$ give $f(T) = p(T)g(T)$ and $g(T)h(T) = h(T)g(T) = I$. So $f(T)X = p(T)X$, which is dense by Lemma 2.2. The last claim follows from Lemma 2.3. $\square$

Proof of Theorem 1.2. Markushevich bases exist in every separable Banach space; see [HVMZ08, Chapter 1]. So there are $e_k \in X$ and $\varphi_k \in X'$ with $\varphi_k(e_j) = \delta_{kj}$, such that $\text{span}\{e_k\}$ is dense and $\{\varphi_k\}$ is total. Replacing (e_k, φ_k) by $(\|\varphi_k\|e_k, \varphi_k/\|\varphi_k\|)$ we may assume $\|\varphi_k\| = 1$. Then $S = \sum_k (k!)^{-1} T^k x_0 \otimes \varphi_k$ with $\sum_k \|T^k x_0\|/k! \leq e^{\|T\|} \|x_0\| < \infty$, so S is nuclear. For $u \in X$ we have $|\varphi_k(u)| \leq \|u\|$, so $f_u(z) = \sum_k \varphi_k(u) z^k/k!$ is entire and $Su = f_u(T)x_0$. If $u \neq 0$, totality gives $\varphi_k(u) \neq 0$ for some k , so $f_u \neq 0$ and $Su \in \text{HC}(T)$ by Lemma 3.1. In particular $Su \neq 0$, so S is injective. Biorthogonality gives $Se_j = T^j x_0/j!$, so $\text{Im } S$ contains the orbit of x_0 and is dense. The last claim is Lemma 2.4. $\square$

4 The space ω

Let $g_1, g_2, \dots$ be the unit vectors of $\varphi = \mathbb{K}^{(\mathbb{N})}$, and $\langle x, g \rangle = \sum_i x_i g_i$ for $x \in \omega$, $g \in \varphi$. We use three standard facts.

(F1) Every continuous linear functional on ω has the form $\langle \cdot, g \rangle$ with $g \in \varphi$, and every linear functional on φ has the form $\langle x, \cdot \rangle$ with $x \in \omega$. The topology of ω is the weak topology $\sigma(\omega, \varphi)$. A base of neighbourhoods of $y \in \omega^m$ is given by the sets $\{z : |z_i^{(a)} - y_i^{(a)}| < \varepsilon, a \leq m, i \leq k\}$.

- (F2) For every linear map $B: \varphi \rightarrow \varphi$ the transpose B^t , defined by $\langle B^t x, g \rangle = \langle x, Bg \rangle$, belongs to $L(\omega)$: the i -th coordinate of $B^t x$ is $\langle x, Bg_i \rangle$, a finite linear combination of coordinates of x . Conversely every $T \in L(\omega)$ equals $(T')^t$, where $T'g = g \circ T$ maps φ into φ by (F1).
- (F3) $\text{Im } B^t = (\ker B)^\perp = \{x : \langle x, g \rangle = 0 \text{ for all } g \in \ker B\}$. In particular $\text{Im } B^t$ is closed, and B^t is injective if and only if B is surjective.

For (F3): clearly $\text{Im } B^t \subseteq (\ker B)^\perp$. If $x \in (\ker B)^\perp$, then $Bg \mapsto \langle x, g \rangle$ is a well-defined linear functional on $\text{Im } B$. Extend it linearly to φ ; by (F1) the extension is $\langle y, \cdot \rangle$ for some $y \in \omega$, and $B^t y = x$. If B is surjective and $B^t y = 0$, then y vanishes on $\text{Im } B = \varphi$, so $y = 0$. If B is not surjective, a nonzero functional on φ that vanishes on $\text{Im } B$ gives $0 \neq y \in \ker B^t$.

We write $A = T'$. Then $\langle T^k x, g \rangle = \langle x, A^k g \rangle$ and, for every polynomial p , $\langle p(T)x, g \rangle = \langle x, p(A)g \rangle$.

Lemma 4.1. *Let $A: \varphi \rightarrow \varphi$ be linear, with $p(A)$ injective for every nonzero $p \in \mathbb{K}[t]$. Let $V, W \subseteq \varphi$ be finite-dimensional. Then there is d such that for every $n > d$ the map A^n is injective on V and $A^n V \cap W = \{0\}$.*

This is a special case of Shkarin's Lemma 1.5 in [Shk11], applied to $L = V + W$, with his Lemma 2.1 for the injectivity of A^n . The module viewpoint goes back to Herzog and Lemmert [HL93], and Shkarin notes that the idea of his proof is close to theirs. We give a short proof.

Proof. Make φ a $\mathbb{K}[t]$ -module, with t acting as A . The submodule M generated by $V + W$ is finitely generated, and it is torsion-free by hypothesis. A finitely generated torsion-free module over the principal ideal domain $\mathbb{K}[t]$ is free. So there are $b_1, \dots, b_r \in M$ such that every $y \in M$ has a unique representation $y = \sum_i p_i(A)b_i$ with $p_i \in \mathbb{K}[t]$. Put $\deg y = \max_i \deg p_i$, with $\deg 0 = -\infty$, measured in this fixed basis. Then $\deg(A^n y) = n + \deg y$ for $y \neq 0$, and $\deg(y + y') \leq \max(\deg y, \deg y')$. Let $d \geq 0$ be at least the maximum of $\deg w$ over a basis of W ; then $\deg w \leq d$ for every $w \in W$. Every A^n is injective. For $v \in V \setminus \{0\}$ and $n > d$ we get $\deg(A^n v) \geq n > d$, so $A^n v \notin W$. $\square$

Proof of Theorem 1.1. (a) $\Rightarrow$ (b). Let $p \neq 0$. If $p(A)g = 0$ with $g \neq 0$, then $\langle p(T)x, g \rangle = 0$ for all x , so $p(T)$ does not have dense range. This contradicts Lemma 2.2.

(c) $\Rightarrow$ (a) is the case $n = 1$.

(b) $\Rightarrow$ (c). Fix a countable dense set $D \subseteq \mathbb{K}$ with $1 \in D$. Let $\mathcal{R}$ be the set of all triples $\rho = (J, k, \Lambda)$, where $J = (j_1, \dots, j_m)$ is an m -tuple of distinct positive integers for some $m \geq 1$, $k \geq 1$, and $\Lambda = (\Lambda_{il}) \in D^{k \times m}$. The set $\mathcal{R}$ is countable. Enumerate it as $\rho_1, \rho_2, \dots$, with $\rho_s = (J_s, k_s, \Lambda^{(s)})$ and $J_s = (j_1^{(s)}, \dots, j_{m_s}^{(s)})$. Let $V_k = \text{span}\{g_1, \dots, g_k\}$.

Step 1: times. Put $W_0 = \{0\}$. Given the finite-dimensional space W_{s-1} , Lemma 4.1 gives n_s such that A^{n_s} is injective on V_{k_s} and $A^{n_s} V_{k_s} \cap W_{s-1} = \{0\}$. Put $W_s = W_{s-1} \oplus A^{n_s} V_{k_s}$. By induction the family $\mathcal{F} = \{A^{n_s} g_i : s \geq 1, 1 \leq i \leq k_s\}$ is linearly independent: its members with index at most s form a basis of W_s .

Step 2: the map B . Extend $\mathcal{F}$ to a basis of φ . Define a linear map $B: \varphi \rightarrow \varphi$ by

$$B(A^{n_s} g_i) = \sum_{l=1}^{m_s} \Lambda_{il}^{(s)} g_{j_l^{(s)}} \quad (s \geq 1, 1 \leq i \leq k_s),$$

and by $B = 0$ on the remaining basis vectors. For each j the triple $((j), 1, (1))$ occurs as some ρ_s , and then $B(A^{n_s} g_1) = g_j$. So B is surjective. Put $S = B^t$. By (F2) and (F3), $S \in L(\omega)$ is injective and $\text{Im } S$ is closed.

Step 3: property (vi). Let $u^{(1)}, \dots, u^{(m)} \in \omega$ be linearly independent. The columns $(u_j^{(1)}, \dots, u_j^{(m)}) \in \mathbb{K}^m$, $j \geq 1$, span $\mathbb{K}^m$. Otherwise some $0 \neq \lambda \in \mathbb{K}^m$ is orthogonal to all of them, and then $\sum_a \lambda_a u^{(a)} = 0$. So there are distinct $j_1, \dots, j_m$ such that the $m \times m$ matrix

$C = (u_{ji}^{(a)})_{a,l}$ is invertible. Let $J = (j_1, \dots, j_m)$. Let $O \subseteq \omega^m$ be nonempty and open. By (F1) it contains a set $\{z : |z_i^{(a)} - Y_{ai}| < \varepsilon, a \leq m, i \leq k\}$ for some k , some $Y \in \mathbb{K}^{m \times k}$ and some $\varepsilon > 0$. The map $\Lambda \mapsto C\Lambda^\top$ is a linear bijection $\mathbb{K}^{k \times m} \rightarrow \mathbb{K}^{m \times k}$, and $D^{k \times m}$ is dense. So there is $\Lambda \in D^{k \times m}$ with $|(C\Lambda^\top)_{ai} - Y_{ai}| < \varepsilon$ for all a, i . Let s be such that $\rho_s = (J, k, \Lambda)$. For $a \leq m$ and $i \leq k$,

$$(T^{n_s} S u^{(a)})_i = \langle u^{(a)}, B A^{n_s} g_i \rangle = \sum_{l=1}^m \Lambda_{il} u_{jl}^{(a)} = (C\Lambda^\top)_{ai}.$$

So $(T^{n_s} S u^{(a)})_{a \leq m} \in O$. Hence $(S u^{(1)}, \dots, S u^{(m)})$ is hypercyclic for $T^{\oplus m}$.

The remaining statements: $\text{Im } S$ is closed by (F3) and infinite-dimensional because S is injective. Since S is injective, every linearly independent tuple in $\text{Im } S$ is the image of a linearly independent tuple. The case $n = 1$ shows $\text{Im } S \setminus \{0\} \subseteq \text{HC}(T)$. The statement about L_T is Lemma 2.5. $\square$

The mechanism of Step 3, prescribing finitely many pairings with linearly independent functionals of the form $A^n g$, is the one in Shkarin's proof of [Shk11, Theorem 1.1]. Here the pairings are prescribed once and for all through the map B , so that a single operator S serves all linearly independent tuples.

Remark 4.2 (Menet's criterion and his Remark 4.10). Menet [Men13, Section 4.2] studies sequences (T_k) of operators $\omega \rightarrow \omega$, indexing coordinates from 0, and writes T_k as a matrix $(a_{i,j}^{(k)})$ whose j -th column is $T_k e_j$. His Theorem 4.8 states that (T_k) has a hypercyclic subspace if, for all $N \geq 0$, $l \geq 1$ and $K \geq 1$, there is $k \geq K$ with $\text{span}\{(a_{i,j}^{(k)})_{i < l} : j \geq N\} = \mathbb{K}^l$. He gives a sequence with a hypercyclic subspace for which this hypothesis fails. In [Men13, Remark 4.10] he asks whether, for an operator and its iterates, the hypothesis is equivalent to having a hypercyclic subspace. For $T_k = T^k$, the i -th row of the matrix of T^k is $A^k g_i$. So, in our indexing from 1, the hypothesis for given (N, l, k) says that no nonzero combination of $A^k g_1, \dots, A^k g_l$ lies in $\text{span}\{g_1, \dots, g_N\}$. If T is hypercyclic, this holds for all large k by Lemma 4.1, applied to $V = \text{span}\{g_1, \dots, g_l\}$ and $W = \text{span}\{g_1, \dots, g_N\}$. Hence, for $T \in L(\omega)$ and its iterates, the following are equivalent: the hypothesis of [Men13, Theorem 4.8]; the existence of a hypercyclic subspace; hypercyclicity. This answers Menet's question positively in this case. It also shows that the hypercyclic-subspace part of Theorem 1.1 follows from [Shk11, Lemma 1.5] and [Men13, Theorem 4.8]. We did not find this consequence stated in the literature. It does not give property (vi). Menet also gives a general criterion for the absence of hypercyclic subspaces of type 2 [Men13, Theorem 3.6], that is, of hypercyclic subspaces M such that $M \cap \ker p$ is infinite-dimensional for every continuous seminorm p , and he applies it to universal series [Men13, Theorem 4.2]. This does not conflict with Theorem 1.1. On ω every continuous seminorm has a kernel of finite codimension, so every hypercyclic subspace is of type 2. By Theorem 1.1, the hypothesis of [Men13, Theorem 3.6] therefore fails for the iterates of every hypercyclic operator on ω .

Remark 4.3 (Petersson's theorem on ω). By (F2) and (F3), every injective $S \in L(\omega)$ has closed infinite-dimensional range. So an admissible operator for T on ω produces a hypercyclic subspace. Conversely, Petersson [Pet07, Theorem 2] proved that if a sequence of operators on ω has a hypercyclic subspace H , then there is an injective $S \in L(\omega)$ with $\text{Im } S \subseteq H$. His proof is a short triangular construction: S maps the k -th unit vector to a vector of H whose first nonzero coordinate has index n_k , with n_k strictly increasing. For a single operator such an S is admissible. Thus, for a single operator on ω , (Q1) is equivalent to the existence of a hypercyclic subspace, and by Theorem 1.1 both hold for every hypercyclic operator. The paper [Pet07] treats sequences of operators and assumes a hypercyclic subspace. With the theorem of Bès and Conejero it gives admissible operators for $p(B)$, where p is a non-constant polynomial and B is the backward shift [Pet07, Corollary 1]. It does not state that every hypercyclic operator on ω has a hypercyclic subspace, and it does not consider property (vi). Its only discussion of L_T is its account of [Cha99, CT01, BMGP04], recalled in Remark 4.4. It asks whether its Theorem 2 holds on every separable Fréchet space; see (O2) in Section 6.

Remark 4.4 (The operator L_T). By Lemma 2.5, S has property (vi) for T if and only if S is a hypercyclic vector for L_T on $(L(X), \text{SOT})$. As recalled by Petersson [Pet07, Proposition 2], Theorems 3.1 and 3.5 of [BMGP04] give the following. Let X be a separable Fréchet space with a continuous norm, and suppose that the powers of T satisfy the Universality Criterion of [BMGP04, Definition 2.1] along some sequence. Then L_T has a compact SOT-hypercyclic vector. For a single operator we read this hypothesis as the Hypercyclicity Criterion, which is equivalent to weak mixing by [BP99]. For Banach spaces see also Chan and Taylor [CT01, Corollary 6], and Chan [Cha99] for Hilbert spaces. Petersson adds that this does not remain true without a continuous norm [BMGP04, Example 3.2]. On ω no injective operator is compact. Indeed, every 0-neighbourhood in ω contains a subspace of finite codimension, and a compact operator maps some 0-neighbourhood into a relatively compact, hence bounded, set, so it vanishes on such a subspace. So the operators S of Theorem 1.1 are SOT-hypercyclic vectors for L_T that are not compact. We have not read [Cha99, CT01, MGP03, BMGP04]. By its zbMATH review, [MGP03, Theorem 2.3] extends the theorem on L_T from Banach spaces to locally convex spaces. We did not check whether its hypotheses allow spaces without a continuous norm, such as ω . If they do, then property (vi) on ω follows from it together with [BM07] and Lemma 2.5, and the implication (a) $\Rightarrow$ (c) of Theorem 1.1 is not new.

5 Rank-one perturbations

Proof of Theorem 1.3. The identity $S' = S_0 + (TS_0a - S_0e) \otimes \psi$ is immediate. Since $\psi(a) = 0$ and $\psi(e) = 1$, we get $S'a = S_0a = x$ and $S'e = TS_0a = Tx$. Let $u \neq 0$ and $w = u - \psi(u)e \in \ker \psi$. There are three cases.

- $\psi(u) = 0$. Then $S'u = S_0u \in \text{HC}(T)$ by (vi) with $n = 1$.
- $\psi(u) \neq 0$ and w, a linearly independent. By (vi) with $n = 2$ the pair (S_0w, S_0a) is hypercyclic for $T \oplus T$. The map $P(y, z) = y + \psi(u)Tz$ from X^2 to X is continuous and surjective, and $P(T \oplus T) = TP$. By Lemma 2.3, $S'u = P(S_0w, S_0a) \in \text{HC}(T)$.
- $\psi(u) \neq 0$ and $w = \beta a$ for some $\beta \in \mathbb{K}$. Then $S'u = (\beta I + \psi(u)T)S_0a$. This lies in $\text{HC}(T)$ by Lemma 2.3, since $S_0a \in \text{HC}(T)$ and $\beta + \psi(u)t \neq 0$.

In all cases $S'u \in \text{HC}(T)$, in particular $S'u \neq 0$. So S' is admissible. The vector x is hypercyclic, hence not an eigenvector, so x and Tx are linearly independent. By Lemma 2.4, (x, Tx) is not hypercyclic for $T \oplus T$. $\square$

6 The answer, and what remains open

Proof of Corollary 1.4. (i) The Banach and ω cases are Theorems 1.1 and 1.2. If X has a continuous norm and T is weakly mixing, the source states that there is an operator S with (v). It gives no reference for this statement; its reference [BP99] is attached to the equivalences before it. The statement follows from [BMGP04, Theorems 3.1 and 3.5], as recalled in [Pet07, Proposition 2], together with [BP99] (Remark 4.4). We have not read [BMGP04], and we use this input without proof in (i)–(iii). This S has property (vi) by Lemma 2.5. It is injective, because Sx is hypercyclic for every $x \neq 0$, so it is admissible.

(ii) On Banach spaces, Theorem 1.2 gives, for every hypercyclic T , an admissible S with the non-hypercyclic pair (x_0, Tx_0) in its range. In the other two cases of (i), T is weakly mixing, and the operators with property (vi) from (i) and from Theorem 1.1, fed into Theorem 1.3, give an admissible S' with the non-hypercyclic pair (x, Tx) in its range. (If T is not weakly mixing, every admissible S is a witness: its range is infinite-dimensional, and no pair is hypercyclic for $T \oplus T$.)

(iii) If S is as in (Q2'), then $\text{Im } S$ contains two linearly independent vectors, and they form a hypercyclic vector for $T \oplus T$. So T is weakly mixing. Conversely, if T is weakly mixing, the operators with property (vi) from (i) and from Theorem 1.1 work, since they are injective. On ω every hypercyclic operator is weakly mixing. This follows from [BM07]; from the fact that every hypercyclic operator on ω is mixing [Shk15, Theorem 1.7], [Shk11, Corollary 1.2]; or from Theorem 1.1 with $n = 2$. On Banach spaces, weak mixing is equivalent to the Hypercyclicity Criterion [BP99]. $\square$

Thus (Q2) is negative whenever (Q1) is known to be positive. The following cases of (Q1) are not settled here.

- (O1) X is a non-normable Fréchet space with a continuous norm and T is hypercyclic but not weakly mixing. Such operators exist, for instance on the space $H(\Omega)$ of holomorphic functions on a simply connected domain $\Omega \subseteq \mathbb{C}$ [BM07]. Our proof of Theorem 1.2 uses a Banach norm, through the entire functional calculus.
- (O2) X has no continuous norm and X is not isomorphic to ω , for example $X = \omega \times \ell^2$. We do not settle this case even for mixing T . We do not know whether operators with property (vi) exist here, and ranges of operators need not be closed. Hypercyclic subspaces cannot always help: an example of Bonnet, Martínez-Giménez and Peris [BMGP04], recalled in [Men13, Section 1], is a mixing operator on such a space without a hypercyclic subspace (see also [Pet07, p. 1146]). When a hypercyclic subspace exists, (Q1) would follow from a positive answer to Petersson's question [Pet07, p. 1147] whether his Theorem 2 holds on every separable Fréchet space.

For hereditarily hypercyclic, that is, weakly mixing, operators, as in the corpus record, the only case of (Q1) settled neither here nor in the source is (O2); case (O1) concerns operators that are not weakly mixing.

Verification. All proofs in this note are by hand; no statement depends on a computation. Three sets of finite checks, in exact arithmetic unless stated otherwise, were run as sanity checks of the bookkeeping.

1. A program by the finder, `omega_construction.py` (standard library, exact rational arithmetic, under a minute), implements the construction in the proof of Theorem 1.1 over $\mathbb{K} = \mathbb{Q}$. It treats three torsion-free examples: A the forward shift on φ (so T is the backward shift), $A = (I + \text{shift})^2$, and multiplication by t on the localisation $\mathbb{K}[t, 1/(t-1)]$, which is not a free module. It runs 112 requirements ρ_s , 88 of them for pairs. For each example it checks that every requirement found a valid time n_s . It computes the first k coordinates of $T^{n_s}Su$ by iterating the row-finite matrix of T on coordinates, and compares them with the prescribed values and with the dual formula in Step 3; in each example all 784 comparisons agree. It also checks that (Su, Su') , for the first two unit vectors u, u' of ω , attains all 80 prescribed 2×2 patterns. A second program, `identities.py`, checks the rank-one identities used in Theorem 1.3 on 200 random rational instances. Its two further checks, of $Se_j = T^j x_0 / j!$ and of the graph relation behind Lemma 2.4, are tautological as written: each compares an expression with itself.
2. An independent referee wrote `indep_check.py` from the written proofs, with a different algorithm: one global row reduction, an explicit row-finite matrix for S , and primal iteration of T only. It covers the backward shift, an upper staircase matrix with non-constant coefficients, and multiplication by t on $\mathbb{K}[t, 1/t]$. It confirms the coordinate identity of Step 3 in 250 cases per example. For one fixed linearly independent triple and six random targets per example, it solves $\Lambda = (C^{-1}Y)^\top$ exactly and hits every target exactly ($18 = 3 \times 6$ hits per example). It checks that the bound of Lemma 4.1 is sharp for the shift, that two examples with torsion fail for every $n \leq 60$, and the identities of Theorem 1.3 on these instances.

3. A second referee wrote `r2_omega_check.py` from the paper alone, before reading the other scripts (standard library, exact rational arithmetic, about ten seconds). It uses seven examples, five torsion-free (among them multiplication by t on $\mathbb{K}[t, 1/(t^2 + 1)]$) and two with torsion. On all of them it checks that row i of the matrix of T^k is $A^k g_i$, the conclusion of Lemma 4.1, and the translation of Menet’s hypothesis in Remark 4.2. The bound of Lemma 4.1 is sharp for two examples, and the examples with torsion fail for every $n \leq 60$. On the torsion-free examples it runs the construction of Theorem 1.1 with 66 requirements each, iterates T on Su for vectors u that are not finitely supported, and hits random targets exactly for 18 random linearly independent tuples per example. It also checks the identities of Theorems 1.2 and 1.3 on random instances, and, in floating point, the factorisation in the proof of Lemma 3.1 for $f(z) = e^z - 1$.

Both referees also checked every proof by hand, and the second referee checked the source and the literature. The scripts and their outputs are included in the source archive.

Scope and priority. The note answers (Q1) positively on separable Banach spaces and on ω . For weakly mixing operators on spaces with a continuous norm, (Q1) is already answered in the source; we use this input without proof (Corollary 1.4). The note answers (Q2) negatively in every setting where (Q1) is known to hold. It is a partial answer, since (Q1) is not settled in the cases (O1) and (O2); for hereditarily hypercyclic operators only (O2) remains. We read the source [Pet06] in the original report, the full texts of [Pet07, Men13, Shk11], and the relevant parts of [Shk15, BS24]. We could not access [Sal11]. According to its zbMATH review it ends with a list of open questions, and we do not know whether Theorem 1.1 answers one of them. What we say about [HL93, BM07, BC06, BP99, DR09, Cha99, CT01, MGP03, BMGP04] is taken from their zbMATH reviews and from [Pet06, Pet07, Shk11, Shk15, Men13, BS24]. In particular, we did not check whether [MGP03, Theorem 2.3] applies to spaces without a continuous norm; if it does, property (vi) on ω is not new (Remark 4.4). We searched zbMATH, Crossref, arXiv and Semantic Scholar, and made three web searches. We also looked at the later papers that cite [Men13] or [Pet07] according to Semantic Scholar, through their abstracts and, where available, their full texts. We found no statement that every hypercyclic operator on ω has a hypercyclic subspace or an operator with property (vi). Only the first of these statements follows quickly from [Shk11] and [Men13] (Remark 4.2), so it may be known to specialists. The statement about property (vi) does not follow directly from these two papers. This negative search is not a proof of priority. The refutation of (Q2) is elementary, and Theorem 1.2 is folklore.

AI-assisted tools were used to search the problem corpus and the literature, to find the arguments, to write and run the verification programs, and to help prepare this manuscript. The author has reviewed the proofs, the computations and the sources and is responsible for all claims. The note has not been peer reviewed.

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