

Conformal Homeomorphisms of Strongly Pseudoconvex CR 3-Manifolds Are CR or Anti-CR: On a Question of Puqi Tang

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Abstract

Puqi Tang asked in the 1995 Several Complex Variables Problem List whether a homeomorphism between strongly pseudoconvex CR 3-manifolds that maps infinitesimal spheres of a Carnot–Carathéodory metric to infinitesimal spheres is CR; for this metric reading the answer is *no* as literally posed, because complex conjugation of the sphere $S^3 \subset \mathbb{C}^2$ is such a map and is anti-CR, and *yes* under the convention of Korányi and Reimann that the map preserves the coorientation of the contact planes ($f^*\theta' = \lambda\theta$ with $\lambda > 0$). More precisely, modulo Theorem 1.1 of Capogna, Citti, Le Donne and Ottazzi (J. Math. Pures Appl. 2019), a published theorem used without re-proof, we show the following. Let the Carnot–Carathéodory distances come from smooth Hermitian metrics on the contact planes, and let the metric distortion of the homeomorphism f be 1 at every point. Then f is a C^∞ diffeomorphism which on each connected component is CR or anti-CR, and it is CR exactly where it preserves the coorientation of the contact planes. Conversely, every such diffeomorphism has distortion 1 for every choice of Hermitian metrics. The manifolds need not be compact, connected or embeddable. The whole analytic content, the smoothness of 1-quasiconformal maps between sub-Riemannian contact manifolds, is Theorem 1.1 of Capogna, Citti, Le Donne and Ottazzi; the answer is a corollary of that theorem and of elementary linear algebra, which for the flat model is already in the work of Cowling, Li, Ottazzi and Wu (2020); no new technique is claimed, and the novelty is low. Tang (1996) had proved the CR conclusion for locally embeddable manifolds and conformal homeomorphisms with locally integrable horizontal derivatives. The question is thus answered for the Carnot–Carathéodory metric reading, as a corollary of the 2019 theorem; the analytic reading of Tang (1996), in which conformality is defined through differentials that exist almost everywhere, remains open. This is an unrefereed note.

1 Introduction

The answer to Puqi Tang’s question below, for the reading of “conformal” through the Carnot–Carathéodory metric, is *no* as literally posed and *yes* under the convention of Korányi and Reimann that the map preserves the coorientation of the contact planes ($f^*\theta' = \lambda\theta$ with $\lambda > 0$); this is a corollary of [CCLO19, Theorem 1.1], a published theorem used without re-proof, and of elementary linear algebra.

1.1 The question

In the Several Complex Variables Problem List of September 1995 [BFGK95] (Problem 2.0.3 in Section 2, “New problems in this edition”, repeated as Problem 12.3.1 in Subsection 12.3, “Quasiconformal mappings”, in the numbering of the typeset list; the list’s own cross-reference calls this subsection

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“Section 11.3”), Puqi Tang, under the heading “Liouville type assertion on CR 3-manifolds”, recalls that a Carnot–Carathéodory metric can always be defined on a CR 3-manifold, calls a homeomorphism between two strongly pseudoconvex CR 3-manifolds *conformal* if it maps infinitesimal spheres with respect to a Carnot–Carathéodory metric to infinitesimal spheres, and asks: “Is a conformal homeomorphism CR?” The question is the record AMR-119-0003 of the UnsolvedMath dataset [ula26]; the record AMR-119-0078 has the same text. Both are labelled open.

We read the question as follows (Section 2). The Carnot–Carathéodory distances d, d' are those of smooth Hermitian metrics g, g' on the contact planes, and f is conformal if its metric distortion

$$H_f(p) = \limsup_{r \rightarrow 0} \frac{\sup\{d'(f(p), f(q)) : d(p, q) \leq r\}}{\inf\{d'(f(p), f(q)) : d(p, q) \geq r\}}$$

equals 1 at every point p . The companion problem in [BFGK95] measures the distortion of quasiconformal diffeomorphisms of strongly pseudoconvex CR manifolds with Hermitian metrics on the contact bundles, which supports this reading of “Carnot–Carathéodory metric”; by Lemma 3.1, spheres $d(p, q) = r$ in place of balls give the same notion. The answer depends on an orientation convention, which the question does not state.

1.2 Result

Theorem 1.1. *Let (M, H, J) and (M', H', J') be strongly pseudoconvex CR 3-manifolds of class C^∞ , not necessarily compact, connected or embeddable. Let g, g' be smooth Hermitian metrics on H, H' with Carnot–Carathéodory distances d, d' , and let $f: M \rightarrow M'$ be a homeomorphism with $H_f(p) = 1$ for every $p \in M$. Then, modulo [CCLO19, Theorem 1.1], a published theorem used without re-proof:*

- (1) *f is a C^∞ diffeomorphism, $df(H) = H'$, and for every p the restriction of df_p to H_p is a similarity from (H_p, g_p) onto $(H'_{f(p)}, g'_{f(p)})$;*
- (2) *on each connected component of M , f is CR or anti-CR;*
- (3) *if θ, θ' are adapted contact forms (Definition 2.2) near p and $f(p)$ and $f^*\theta' = u\theta$ near p , then f is CR near p if and only if $u(p) > 0$, and anti-CR near p if and only if $u(p) < 0$; thus f is CR exactly on the components of M on which it preserves the canonical coorientation of the contact planes.*

Conversely:

- (4) *every C^1 diffeomorphism $f: M \rightarrow M'$ that is CR or anti-CR on each component of M satisfies $H_f = 1$ everywhere, for every choice of smooth Hermitian metrics g, g' .*

Corollary 1.2 (answer to the question). *In the setting of Theorem 1.1:*

- (a) *As literally posed, the answer is no: complex conjugation $\sigma(z, w) = (\bar{z}, \bar{w})$ of the sphere $S^3 \subset \mathbb{C}^2$ and the map $\tau(x, y, t) = (x, -y, -t)$ of the Heisenberg group are isometries of Carnot–Carathéodory distances of Hermitian metrics, hence conformal, and they are anti-CR, hence CR at no point (Section 5).*
- (b) *Every conformal homeomorphism that preserves the coorientation of the contact planes is CR. In particular the answer is yes under the convention $f^*\theta' = \lambda\theta$ with $\lambda > 0$ for the contact forms with positive Levi forms, which is the convention of [KR90] as described in its zbMATH review.*
- (c) *Equivalently, on each component of M a conformal homeomorphism is CR as a map $(M, J) \rightarrow (M', J')$ or as a map $(M, J) \rightarrow (M', -J')$.*

(d) *Conformality does not depend on the choice of the Hermitian metrics g, g' .*

Proof. (a) is Propositions 5.1 and 5.2. (b) follows from Theorem 1.1(3), since $u > 0$ everywhere for such a map. (c) holds because a map is anti-CR for J' if and only if it is CR for $-J'$. (d) If f is conformal for g, g' , then by Theorem 1.1(1)–(2) it is a C^∞ diffeomorphism which is CR or anti-CR on each component, so by Theorem 1.1(4) it is conformal for any other Hermitian metrics. $\square$

By Lemma 3.1, the hypothesis $H_f = 1$ may be replaced by the sphere version $H_f^\pm = 1$ everywhere. Since [CCLO19, Theorem 1.1] needs only a uniform bound $H_f \leq K$ together with $H_f = 1$ almost everywhere, Theorem 1.1 also holds under this weaker hypothesis; we use the hypothesis “at every point”, which is what the words of the question express.

1.3 What is known and what is new

Tang [Tan96, Theorem 2.3] proved, before the problem list appeared (arXiv, December 1993), the prior partial result: if M_1, M_2 are smooth, strongly pseudoconvex and locally embeddable, and f is a conformal homeomorphism in the analytic sense of his Definition 2.1 with L_{loc}^1 horizontal derivatives, then f is smooth and CR. The abstract of his thesis [Tan94] states the same result; the thesis itself, where the conventions may be fixed, was not read. The question of the problem list asks for the case without embeddability and without a priori Sobolev regularity. A neutral remark: the proof of [Tan96, Theorem 2.3] uses the formula $K(f) = (1 + |\mu|)/(1 - |\mu|)$ (his formula (2.9)), which presupposes $|\mu| < 1$, that is, that the differential preserves the orientation of the contact planes; complex conjugation of S^3 (Proposition 5.1) is smooth, conformal and anti-CR between embeddable manifolds, so the statement needs this implicit convention. The companion problem of [BFGK95] speaks of quasiconformal maps that distort the CR structures boundedly, which in Tang’s framework means a Beltrami coefficient with $|\mu| < 1$; this supports the reading of the question with the orientation convention, under which the answer is a plain yes.

Korányi and Reimann proved that C^4 conformal maps of the Heisenberg group are smooth and CR ([KR85, Theorem 8], as quoted in [Tan96]); in [KR90] quasiconformal maps of strictly pseudoconvex CR manifolds are C^2 contact transformations with $f^*\theta' = \lambda\theta$, $\lambda > 0$. We have not read [KR85, KR90]; the convention is taken from the zbMATH review of [KR90]. According to Wu and Wang [WW12, Section 1], Korányi and Reimann [KR95] proved that 1-quasiconformal maps of the Heisenberg group are CR or anti-CR; we have not read [KR95].

The whole analytic content of Theorem 1.1 is the theorem of Capogna, Citti, Le Donne and Ottazzi [CCLO19, Theorem 1.1]: every 1-quasiconformal map between sub-Riemannian contact manifolds is conformal, that is, a smooth diffeomorphism whose differential maps horizontal vectors to horizontal vectors and is a similarity on the horizontal spaces. The paper does not discuss CR or anti-CR maps and does not mention the problem list; the published version cites [Tan96] twice, in a list of works on 1-quasiconformal maps of Carnot groups and, after Theorem 1.1, for related results in the setting of CR 3-manifolds (the arXiv version v2 cites it three times). Cowling, Li, Ottazzi and Wu [CLOW20, Theorem 3.3] proved, for a tight stratified group (by their Corollary 3.2, the Heisenberg groups and certain groups with two generators) with the Carnot distance determined by the Levi form, that a homeomorphism of a connected open subset is 1-quasiconformal if and only if it is CR or anti-CR; their proof states the linear algebra of Lemma 4.1 (numbers of the arXiv version v2). This is the flat model; general CR 3-manifolds are not treated there. Earlier, Wu and Fu [WF14] had shown that 1-quasiconformal maps on Goursat groups are CR or anti-CR (we read only the abstract). Wu and Wang [WW12, Corollary 1.1] proved that an orientation-preserving (orientation-reversing) 1-quasiconformal map between domains of a smooth strongly pseudoconvex hypersurface of $\mathbb{C}^{n+1}$ is CR (anti-CR) almost everywhere; here orientation refers to the sign of the factor $\gamma(p)$ by which the hc-differential of the map multiplies the vertical direction, transverse to the contact planes [WW12,

after Proposition 3.1]. Capogna and Le Donne [CL21] showed that boundary extensions of isometries between strongly pseudoconvex domains are 1-quasiconformal, so that [CCLO19, Theorem 1.1] gives their smoothness (as described in [CCLO19, Section 1.1]; of [CL21] we read only the arXiv abstract). Falbel and Veloso [FV16] proved, under a different hypothesis, that a diffeomorphism between CR 3-manifolds which preserves $\mathbb{R}$ -circles is CR or anti-CR (we read only the zbMATH review). The Carnot–Carathéodory distance is finite and induces the manifold topology by the theorem of Chow and Rashevskii (see [Cho40] and [CCLO19, Section 2.1]).

What this note adds is only the explicit statement of Theorem 1.1 for general (possibly non-embeddable, non-compact) strongly pseudoconvex CR 3-manifolds, the identification of the sign with the coorientation of the contact planes, and the explicit anti-CR counterexample to the literal question. No printed answer to Tang’s question was found in the sources searched (see “Scope and priority” in Section 6); a search that finds nothing is not a proof of novelty, and no priority is claimed.

2 Definitions and conventions

All manifolds are Hausdorff and second countable, and all objects are of class C^∞ unless said otherwise.

CR 3-manifolds. A *CR 3-manifold* (M, H, J) is a smooth 3-manifold M with a smooth rank-2 subbundle $H \subset TM$ and a smooth bundle endomorphism J of H with $J^2 = -1$. (In this dimension the integrability condition is automatic, since the bundle of $(1, 0)$ -vectors has rank one.) Locally $H = \ker \theta$ for a nowhere vanishing 1-form θ , a *local contact form*; M is *strongly pseudoconvex* if the Levi form is nondegenerate, which in CR dimension one means that $d\theta$ restricted to H_p is nonzero for every p , i.e. that H is a contact distribution. For a local contact form θ put $L_\theta(X, Y) = d\theta(X, JY)$ for $X, Y \in H$.

Lemma 2.1 (adapted forms). *Let M be a strongly pseudoconvex CR 3-manifold and θ a local contact form on a connected open set U .*

- (a) *The sign of $d\theta(X, JX)$ is the same for all $p \in U$ and all nonzero $X \in H_p$.*
- (b) *L_θ is symmetric and J -invariant; it is positive definite if $d\theta(X, JX) > 0$ for $X \neq 0$.*
- (c) *If θ_1, θ_2 are local contact forms with $d\theta_i(X, JX) > 0$ for $X \neq 0$, then $\theta_2 = v\theta_1$ on the intersection of their domains with $v > 0$.*

Proof. (a) For $X \in H_p \setminus \{0\}$ the vectors X, JX are linearly independent, because J has no real eigenvalue; since $d\theta_p$ is a nonzero 2-form on the plane H_p , $d\theta(X, JX) \neq 0$. The function $X \mapsto d\theta(X, JX)$ is continuous and nowhere zero on the connected set $H_p \setminus \{0\}$, so its sign does not depend on X ; evaluating it on a nowhere vanishing local section of H shows that the sign is locally constant in p , hence constant on U . (b) Every 2-form ω on a plane satisfies $\omega(JX, JY) = \det(J)\omega(X, Y)$, and $\det J = 1$ since the eigenvalues of J are $\pm i$. Hence $L_\theta(Y, X) = d\theta(Y, JX) = d\theta(JY, J^2X) = -d\theta(JY, X) = L_\theta(X, Y)$ and $L_\theta(JX, JY) = d\theta(JX, J^2Y) = d\theta(X, JY) = L_\theta(X, Y)$; positivity is $L_\theta(X, X) = d\theta(X, JX)$. (c) Both forms have kernel H , so $\theta_2 = v\theta_1$ with v smooth and nowhere zero. On $H \times H$ we have $d\theta_2 = dv \wedge \theta_1 + v d\theta_1 = v d\theta_1$, hence $v d\theta_1(X, JX) = d\theta_2(X, JX) > 0$ and $v > 0$. $\square$

Definition 2.2. A local contact form θ is *adapted* if $d\theta(X, JX) > 0$ for all nonzero $X \in H$. The *canonical coorientation* of H at p is the open half-space $\{\theta_p > 0\}$ of T_pM for an adapted θ near p .

By Lemma 2.1(a), θ or $-\theta$ is adapted near each point; by (c) the canonical coorientation is well defined. So H is always coorientable, and a partition of unity gives a global adapted form: if $\theta = \sum_i \varphi_i \theta_i$ with adapted θ_i , then θ_p is a positive multiple of each $\theta_{i,p}$ with $\varphi_i(p) > 0$, and $d\theta = \sum_i \varphi_i d\theta_i$ on $H \times H$. Replacing J by $-J$ replaces the adapted forms by their negatives and reverses the canonical coorientation.

Hermitian metrics and the Carnot–Carathéodory distance. A *Hermitian metric* on H is a smooth positive definite fibre metric g with $g(JX, JY) = g(X, Y)$; for example L_θ for an adapted θ . If g is Hermitian and $e \in H_p$ is a unit vector, then (e, Je) is an orthonormal basis of H_p , since $g(e, Je) = g(Je, J^2e) = -g(e, Je)$. Any two Hermitian inner products on H_p are proportional: in a g -orthonormal basis (e, Je) , a second one has $g'(Je, Je) = g'(e, e)$ and $g'(e, Je) = 0$ by the same computation.

A *horizontal curve* is an absolutely continuous curve $\gamma: [a, b] \rightarrow M$ with $\dot{\gamma}(s) \in H_{\gamma(s)}$ for almost every s ; its length is $\ell_g(\gamma) = \int_a^b |\dot{\gamma}(s)|_g ds$. The *Carnot–Carathéodory distance* $d(p, q)$ is the infimum of the lengths of horizontal curves from p to q (and $+\infty$ if there is none). On a connected strongly pseudoconvex M it is finite and induces the manifold topology, by the theorem of Chow and Rashevskii [Cho40], [CCLO19, Section 2.1]. Since restrictions of horizontal curves are horizontal, $d(p, \gamma(s)) \leq \ell_g(\gamma|_{[a, s]})$ for every horizontal γ starting at p .

Remark 2.3 (the same distance as in [CCLO19]). In [CCLO19, Section 2.1] the distance is the infimum of the numbers δ for which there is a C^∞ curve $\gamma: [0, 1] \rightarrow M$ from p to q with $\dot{\gamma} \in H$ and $|\dot{\gamma}|_g \leq \delta$. This is the same distance. Such a curve has length at most δ ; conversely, an absolutely continuous horizontal curve can be reparametrised by arc length and approximated by smooth horizontal curves of almost the same length whose endpoints are close to the given ones (approximate its coefficients in a local orthonormal frame), the endpoints can be joined by short smooth horizontal curves (Chow–Rashevskii), and a smooth horizontal curve of length ℓ can be reparametrised on $[0, 1]$ with speed at most $\ell + \varepsilon$. This is standard; we use it so that the definitions and theorems of [CCLO19] are applied to the same metric spaces.

Metric distortion. For a homeomorphism $f: M \rightarrow M'$, a point $p \in M$ and $r > 0$ put

$$\begin{aligned} L_f(p, r) &= \sup\{d'(f(p), f(q)) : d(p, q) \leq r\}, & \ell_f(p, r) &= \inf\{d'(f(p), f(q)) : d(p, q) \geq r\}, \\ L_f^-(p, r) &= \sup\{d'(f(p), f(q)) : d(p, q) = r\}, & \ell_f^-(p, r) &= \inf\{d'(f(p), f(q)) : d(p, q) = r\}, \end{aligned}$$

and $H_f(p) = \limsup_{r \rightarrow 0} L_f(p, r)/\ell_f(p, r)$, $H_f^-(p) = \limsup_{r \rightarrow 0} L_f^-(p, r)/\ell_f^-(p, r)$. This H_f is the distortion of [CCLO19, Definition 3.2] (arXiv v2: Definition 1.2).

Definition 2.4. A homeomorphism $f: M \rightarrow M'$ between strongly pseudoconvex CR 3-manifolds with Hermitian metrics on H, H' is *conformal* if $H_f(p) = 1$ for every $p \in M$.

This is our reading of “maps infinitesimal spheres to infinitesimal spheres”: for small r the image of the ball (or, by Lemma 3.1, of the sphere) of radius r about p lies between two spheres about $f(p)$ whose radii have ratio tending to 1. By Lemma 3.1, $L_f(p, r) \geq \ell_f(p, r) > 0$ for small r when M is connected, so $H_f \geq 1$, and $\limsup$ may be replaced by $\lim$ in Definition 2.4. Points of other components are at infinite distance and do not affect $H_f(p)$.

CR and anti-CR maps. A C^1 map $f: M \rightarrow M'$ with $df(H) \subset H'$ is *CR* if $df \circ J = J' \circ df$ on H , and *anti-CR* if $df \circ J = -J' \circ df$ on H . An anti-CR map $(M, J) \rightarrow (M', J')$ is the same as a CR map $(M, J) \rightarrow (M', -J')$, and $(M', H', -J')$ is again a strongly pseudoconvex CR 3-manifold.

3 Two metric lemmas

In this section M, M' are connected strongly pseudoconvex CR 3-manifolds with smooth fibre metrics g, g' on H, H' (Hermitian or not) and Carnot–Carathéodory distances d, d' ; $B(p, r) = \{q : d(p, q) < r\}$ and $S(p, r) = \{q : d(p, q) = r\}$.

Lemma 3.1 (balls versus spheres). *Let $f : M \rightarrow M'$ be a homeomorphism and $p \in M$. There is $r_0 > 0$ such that for $0 < r \leq r_0$ the sphere $S(p, r)$ is nonempty and compact,*

$$\ell_f(p, r) = \ell_f^-(p, r) > 0 \quad \text{and} \quad L_f(p, r) = \sup_{0 < t \leq r} L_f^-(p, t).$$

Consequently $H_f(p) = H_f^-(p)$.

Proof. Write L, ℓ, L^-, ℓ^- for the four functions of r at p . Choose a compact coordinate ball $K \neq M$ around p and $r_0 > 0$ with $B(p, 2r_0) \subset K$; this is possible because d induces the topology. Let $0 < r \leq r_0$ and $U = B(p, r)$. Then $\{d(p, \cdot) \leq r\}$ is a closed subset of K , hence compact, and so is $S(p, r)$.

The sphere is nonempty. U is open, nonempty and not equal to M , and M is connected, so its boundary ∂U is nonempty. As the closure of U lies in the closed set $\{d(p, \cdot) \leq r\}$, we have $\partial U \subset S(p, r)$. Hence $\ell(r) \leq \ell^-(r)$, since $S(p, r)$ is a nonempty subset of $\{d(p, \cdot) \geq r\}$.

The reverse inequality. Let $d(p, q) \geq r$. Then $q \notin U$, so $f(q)$ lies outside the open set $f(U)$, which contains $f(p)$. Let $\sigma : [0, 1] \rightarrow M'$ be a horizontal curve from $f(p)$ to $f(q)$. The set of s with $\sigma(s) \notin f(U)$ is closed, contains 1 and not 0; let $s_0 > 0$ be its least element. Then $\sigma(s_0)$ lies in the closure of $f(U)$ but not in $f(U)$, that is, in $\partial f(U) = f(\partial U) \subset f(S(p, r))$. By the definition of d' as an infimum of lengths of horizontal curves (restrictions of horizontal curves are horizontal),

$$\ell_{g'}(\sigma) \geq \ell_{g'}(\sigma|_{[0, s_0]}) \geq d'(f(p), \sigma(s_0)) \geq \ell^-(r).$$

Taking the infimum over σ gives $d'(f(p), f(q)) \geq \ell^-(r)$, so $\ell(r) \geq \ell^-(r)$. Moreover $\ell^-(r) > 0$: it is the minimum of the continuous function $q \mapsto d'(f(p), f(q))$ on the nonempty compact set $S(p, r)$, and this function is positive there because f is injective.

The supremum. The closed ball $\{d(p, \cdot) \leq r\}$ is the union of $\{p\}$ and of the spheres $S(p, t)$, $0 < t \leq r$, which are nonempty; hence $L(r) = \sup_{0 < t \leq r} L^-(t)$.

The distortions. For $0 < r \leq r_0$ we have $L^-(r) \leq L(r)$ and $\ell^-(r) = \ell(r)$, so $H_f^-(p) \leq H_f(p)$. Conversely let $K_1 > H_f^-(p)$ (there is nothing to prove if $H_f^-(p) = \infty$) and take $r_1 \leq r_0$ with $L^-(t) \leq K_1 \ell^-(t)$ for $0 < t \leq r_1$. The function ℓ is nondecreasing, since the set $\{d(p, \cdot) \geq t\}$ shrinks as t grows. For $0 < t \leq r \leq r_1$ we get $L^-(t) \leq K_1 \ell^-(t) = K_1 \ell(t) \leq K_1 \ell(r)$, and taking the supremum over t gives $L(r) \leq K_1 \ell(r)$. Hence $H_f(p) \leq K_1$, and $H_f(p) = H_f^-(p)$. $\square$

The equality of the two infima is noted, for geodesic metric spaces, in the proof of [CCLO19, Corollary 3.26] (arXiv v2: Corollary 3.9), where $H_f = H_f^-$ is proved at almost every point for quasiconformal maps; Lemma 3.1 gives it at every point and for every homeomorphism.

Lemma 3.2 (the easy direction). *Let $f : M \rightarrow M'$ be a C^1 diffeomorphism with $df(H) \subset H'$ and $|df_q X|_{g'} = c(q)|X|_g$ for all $q \in M$ and $X \in H_q$, where $c(q) > 0$. Then $d'(f(p), f(q))/d(p, q) \rightarrow c(p)$ as $q \rightarrow p$, $q \neq p$, and $H_f(p) = 1$ for every $p \in M$.*

Proof. The function c is continuous: $c(q) = |df_q e(q)|_{g'}$ for a local g -unit section e of H . Fix p and $\varepsilon \in (0, 1)$, and take $\rho > 0$ with $|c - c(p)| \leq \varepsilon c(p)$ on $B(p, 2\rho)$. Let $0 < s = d(p, q) < \rho$ and let γ be a horizontal curve from p to q with $\ell_g(\gamma) < \min(2s, (1 + \varepsilon)s)$. Every point of γ is at distance less than

$2s$ from p , so γ lies in $B(p, 2\rho)$. The curve $f \circ \gamma$ is absolutely continuous (f is locally Lipschitz) and horizontal, because $(f \circ \gamma)' = df(\dot{\gamma}) \in H'$, and

$$\ell_{g'}(f \circ \gamma) = \int |\mathrm{d}f(\dot{\gamma})|_{g'} = \int c(\gamma)|\dot{\gamma}|_g \leq (1 + \varepsilon)c(p) \ell_g(\gamma) \leq (1 + \varepsilon)^2 c(p) d(p, q).$$

So $d'(f(p), f(q)) \leq (1 + \varepsilon)^2 c(p) d(p, q)$ when $d(p, q) < \rho$. Since $\mathrm{d}f_q$ is injective and $\dim H_q = \dim H'_{f(q)} = 2$, we have $\mathrm{d}f_q(H_q) = H'_{f(q)}$, so f^{-1} is a C^1 diffeomorphism with $\mathrm{d}f^{-1}(H') \subset H$ and factor $1/c \circ f^{-1}$. The same argument at $f(p)$, together with the continuity of f , gives $d(p, q) \leq (1 + \varepsilon)^2 c(p)^{-1} d'(f(p), f(q))$ for q near p . Hence the ratio tends to $c(p)$. Consequently, for small r , $L_f^-(p, r) \leq (1 + \varepsilon)^2 c(p)r$ and $\ell_f^-(p, r) \geq (1 + \varepsilon)^{-2} c(p)r$, so $H_f^-(p) \leq (1 + \varepsilon)^4$. By Lemma 3.1 $H_f(p) = H_f^-(p)$; as ε is arbitrary and $H_f \geq 1$, $H_f(p) = 1$. $\square$

4 Proof of the theorem

We refer to the published version of [CCLO19], read in the accepted manuscript self-archived by the University of Jyväskylä; the numbers in arXiv:1603.05548v2, which was also read, are given in brackets.

Step 0 (reduction to connected manifolds). A homeomorphism maps components onto components, $H_f(p)$ depends only on the restriction of f to the component of p , and all conclusions of Theorem 1.1 are local or componentwise. So we may assume M and M' connected.

Step 1 (regularity: the published theorem). With M connected, (M, H, g) is a sub-Riemannian manifold in the sense of [CCLO19, Section 2.1] [arXiv v2: Section 2.1]: a connected smooth manifold with a bracket-generating subbundle and a smooth section of positive definite quadratic forms on it. Indeed, for a local frame X, Y of H and a local contact form θ we have $\theta([X, Y]) = -\mathrm{d}\theta(X, Y) \neq 0$, so $H + [H, H] = TM$; in particular M is equiregular, and since H is a contact distribution, M is a sub-Riemannian contact manifold in the sense of [CCLO19]. By Remark 2.3 the distances are those of [CCLO19]. Since $H_f(p) = 1$ for all p , f is quasiconformal in the sense of [CCLO19, Definition 3.2] [1.2] with $K = 1$, and condition (3.4) [(1.4)] of [CCLO19, Theorem 3.3] [1.3], namely $H_f = 1$ almost everywhere, holds; so f is 1-quasiconformal in the sense of [CCLO19, Definition 3.13] [1.13]. By [CCLO19, Theorem 1.1] [1.19], f is conformal in the sense of [CCLO19, Definition 3.1] [1.1]: a smooth diffeomorphism whose differential maps horizontal vectors into horizontal vectors and whose restrictions to the horizontal spaces are similarities. The smoothness is C^∞ ; it is obtained in [CCLO19, Section 5.1] [Section 5.1], in the proof of Theorem 1.4(i) [1.22(i)], by a bootstrap using the Schauder estimates of Proposition 2.12 [2.12]. Since $\mathrm{d}f_p$ is invertible and both planes are 2-dimensional, $\mathrm{d}f_p(H_p) = H'_{f(p)}$. This is conclusion (1); we write $|\mathrm{d}f_p X|_{g'} = c(p)|X|_g$ for $X \in H_p$, with $c(p) > 0$.

Lemma 4.1. *Let $(V, \langle \cdot, \cdot \rangle)$ and $(V', \langle \cdot, \cdot \rangle')$ be Euclidean planes with orthogonal complex structures J, J' ($J^2 = -1$, J orthogonal), and let $A: V \rightarrow V'$ be linear.*

- (a) *If A is a similarity, $|Av|' = c|v|$ with $c > 0$, then $AJ = J'A$ or $AJ = -J'A$, and not both.*
- (b) *If $A \neq 0$ and $AJ = J'A$ or $AJ = -J'A$, then A is a similarity.*

Proof. (a) $B = A/c$ is a linear isometry, so $T = B^{-1}J'B$ is an orthogonal map of V with $T^2 = -1$. T has no real eigenvalue ($Tv = av$ gives $-v = a^2v$), so it is not a reflection; hence T is a rotation by an angle α with $2\alpha \equiv \pi \pmod{2\pi}$, i.e. one of the two rotations by $\pm\pi/2$. For the same reason J is one of these two rotations, and $-J$ is the other. Hence $T = J$ or $T = -J$, that is, $J'B = BJ$ or $J'B = -BJ$. Both cannot hold, since then $2J'A = 0$ while $J'A$ is invertible. (b) Take a unit vector $e \in V$ and a unit

vector $e' \in V'$; then (e, Je) and $(e', J'e')$ are orthonormal bases. Write $Ae = ae' + bJ'e'$. If $AJ = J'A$, then $AJe = aJ'e' - be'$, and the matrix of A is $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$; if $AJ = -J'A$, then $AJe = -aJ'e' + be'$, and the matrix is $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$. In both cases $|Av| = (a^2 + b^2)^{1/2}|v|$, and $a^2 + b^2 > 0$ because $A \neq 0$. $\square$

Step 2 (the sign at each point). A Hermitian metric makes J orthogonal. By Lemma 4.1(a) applied to $A = df_p|_{H_p}$, for each p there is exactly one $\varepsilon(p) \in \{1, -1\}$ with $df_p \circ J = \varepsilon(p)J' \circ df_p$ on H_p .

Step 3 (the sign is locally constant). The sets $P = \{p : df \circ J - J' \circ df = 0 \text{ on } H_p\}$ and $N = \{p : df \circ J + J' \circ df = 0 \text{ on } H_p\}$ are the zero sets of continuous sections of the bundle $\text{Hom}(H, f^*H')$, hence closed. By Step 2 they are disjoint and cover M , so both are open and closed, and on the connected M one of them is M . This is conclusion (2).

Step 4 (which sign). Let θ' be an adapted contact form near $f(p)$ and θ one near p . The form $f^*\theta'$ vanishes on H , since $df(H) = H'$, and is nowhere zero, since df is invertible; so $f^*\theta' = u\theta$ with u smooth and nowhere zero. On $H \times H$, $f^*d\theta' = d(u\theta) = du \wedge \theta + u d\theta$ restricts to $u d\theta$. For nonzero $X \in H_p$ and $Y = df_p X$,

$$d\theta'(Y, df_p(JX)) = u(p) d\theta(X, JX), \quad d\theta(X, JX) > 0.$$

If f is CR near p , then $df_p(JX) = J'Y$ and the left side is positive, so $u(p) > 0$. If f is anti-CR near p , then $df_p(JX) = -J'Y$ and the left side is negative, so $u(p) < 0$. Since exactly one case occurs, f is CR near p if and only if $u(p) > 0$. Finally $u(p) > 0$ means that df_p maps the half-space $\{\theta_p > 0\}$ onto $\{\theta'_{f(p)} > 0\}$, i.e. that f preserves the canonical coorientation at p . This is conclusion (3).

Step 5 (the converse). Let f be a C^1 diffeomorphism which is CR or anti-CR on each component. Then $df(H) \subset H'$, and by Lemma 4.1(b), applied to $df_q|_{H_q}$ (nonzero, as df_q is invertible), $|df_q X|_{g'} = c(q)|X|_g$ with $c(q) > 0$, for any Hermitian g, g' . By Lemma 3.2, applied on each component, $H_f = 1$ everywhere. This is conclusion (4), and the proof of Theorem 1.1 is complete. $\square$

Remark 4.2 (the orientation of M does not detect anti-CR maps). The 3-form $\theta \wedge d\theta$ is unchanged under $\theta \mapsto -\theta$, so it defines a canonical orientation of a contact 3-manifold, and for every C^2 contact diffeomorphism, $f^*\theta' = u\theta$,

$$f^*(\theta' \wedge d\theta') = u\theta \wedge (du \wedge \theta + u d\theta) = u^2 \theta \wedge d\theta.$$

So every such map preserves the canonical orientations, anti-CR maps included. What distinguishes CR from anti-CR conformal maps is the coorientation of the contact planes, equivalently the orientation of the planes themselves, not the orientation of M .

Remark 4.3 (the analytic reading). Tang [Tan96, Definition 2.1] defines conformality analytically: f is ACL, differentiable almost everywhere with $f_*(H) \subset H'$ where the differential exists, and the horizontal differential is a similarity almost everywhere. If such an f is in addition metrically quasiconformal ($H_f \leq K$ everywhere), then condition (3.6) [(1.6)] of [CCLO19, Theorem 3.3] holds, provided that at the points of differentiability the classical differential restricted to H agrees with the horizontal differential used there; then f is 1-quasiconformal and Steps 1–5 apply. We do not prove that Tang's analytic definition by itself implies metric quasiconformality on general CR 3-manifolds; this variant stays open and is outside the claim of this note.

Remark 4.4 (Hermitian metrics are needed). For a sub-Riemannian metric on H that is not Hermitian the statement fails. On the Heisenberg group of Section 5, let g make X, Y orthonormal, let $R(x, y, t) = (-y, x, t)$ be the rotation by $\pi/2$ about the t -axis, and let $A(x, y, t) = (2x, y/2, t)$. Then $A^*\theta = \theta$, and in the frames (X, Y) the differentials of A and R are $\begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}$ and $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The map $F = A^{-1} \circ R \circ A$ is an isometry of the Carnot–Carathéodory distance of the metric A^*g , hence conformal for it, but dF has the matrix $\begin{pmatrix} 0 & -1/4 \\ 4 & 0 \end{pmatrix}$, which neither commutes nor anticommutes with

$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$; so F is neither CR nor anti-CR. (It is CR for the complex structure $dA^{-1}JdA$, for which A^*g is Hermitian.)

Remark 4.5 (what is used where). Strong pseudoconvexity, i.e. the contact condition, is needed for [CCLO19, Theorem 1.1] and in Step 4; the dimension three enters through Lemma 4.1 (rotations of a plane); the Hermitian property makes J, J' orthogonal. Nothing is assumed about compactness, embeddability or real analyticity. The higher-dimensional case and K -quasiconformal maps with $K > 1$ are not treated.

5 The anti-CR examples

Proposition 5.1. *Let $S^3 = \{(z, w) \in \mathbb{C}^2 : |z|^2 + |w|^2 = 1\}$ with $H_p = T_p S^3 \cap iT_p S^3$ and J the multiplication by i , and let g be the Euclidean metric restricted to H . Then g is Hermitian, the form $\theta_p(v) = \langle ip, v \rangle$ (Euclidean inner product of $\mathbb{C}^2 = \mathbb{R}^4$) is an adapted contact form, and complex conjugation $\sigma(z, w) = (\bar{z}, \bar{w})$ is an isometry of the Carnot–Carathéodory distance d , anti-CR, with $\sigma^*\theta = -\theta$. In particular H_σ is 1 everywhere, for g and, by Theorem 1.1(4), for every Hermitian metric, and σ is CR at no point.*

Proof. In real coordinates $p = (x_1, y_1, x_2, y_2)$ with $z = x_1 + iy_1$, $w = x_2 + iy_2$ we have $\theta = -y_1 dx_1 + x_1 dy_1 - y_2 dx_2 + x_2 dy_2$ and $d\theta = 2(dx_1 \wedge dy_1 + dx_2 \wedge dy_2)$, so $d\theta(v, iv) = 2|v|^2$. On $T_p S^3 = p^\perp$ the kernel of θ_p is $p^\perp \cap (ip)^\perp = H_p$; the plane H_p is spanned by $e_1(p) = (-x_2, y_2, x_1, -y_1)$ and $ie_1(p)$, and $d\theta(X, JX) = 2|X|^2 > 0$ on H , so θ is an adapted contact form. Multiplication by i is a Euclidean isometry, so g is Hermitian. The map σ is linear and orthogonal, $\sigma = \text{diag}(1, -1, 1, -1)$ in these coordinates, so $\sigma(S^3) = S^3$ and $d\sigma = \sigma$; moreover $\sigma(iv) = -i\sigma(v)$. If $v \perp p$ and $v \perp ip$, then $\sigma v \perp \sigma p$ and $\sigma v \perp \sigma(ip) = -i\sigma p$; so σ maps H_p onto $H_{\sigma(p)}$, and $d\sigma \circ J = -J \circ d\sigma$ on H : σ is anti-CR. Further $(\sigma^*\theta)_p(v) = \langle i\sigma p, \sigma v \rangle = \langle -\sigma(ip), \sigma v \rangle = -\theta_p(v)$. Since σ is a Euclidean isometry preserving H , it maps horizontal curves to horizontal curves of the same g -length, and $\sigma^{-1} = \sigma$; hence $d(\sigma p, \sigma q) = d(p, q)$. Then $L_\sigma(p, r) \leq r \leq \ell_\sigma(p, r)$, so $H_\sigma \leq 1$, and $H_\sigma = 1$ by Lemma 3.1. By Lemma 4.1(a) (“not both”), σ is CR at no point. $\square$

The Heisenberg group is $\mathbb{R}^3$ with coordinates (x, y, t) , the contact form $\theta = dt + 2(x dy - y dx)$ and $H = \ker \theta$, spanned by $X = \partial_x + 2y \partial_t$ and $Y = \partial_y - 2x \partial_t$. Its CR structure is the one induced from the Siegel domain $\{(z, w) \in \mathbb{C}^2 : \text{Im } w > |z|^2\}$ through the boundary chart $(z, t) \mapsto (z, t + i|z|^2)$, $z = x + iy$: the vector field $Z = \partial_z + i\bar{z} \partial_t$ is mapped to $\partial_z + 2i\bar{z} \partial_w$, which is tangent to the boundary and of type $(1, 0)$, and $Z = \frac{1}{2}(X - iY)$; hence $JX = Y$ and $JY = -X$. Here $d\theta = 4 dx \wedge dy$ and $d\theta(X, JX) = 4 > 0$, so θ is adapted, and the metric g for which (X, Y) is orthonormal is Hermitian.

Proposition 5.2. *On the Heisenberg group, $\tau(x, y, t) = (x, -y, -t)$ satisfies $d\tau(X) = X \circ \tau$, $d\tau(Y) = -Y \circ \tau$ and $\tau^*\theta = -\theta$. Hence τ is an isometry of the Carnot–Carathéodory distance of g , conformal and anti-CR, and CR at no point.*

Proof. τ is linear with $d\tau = \text{diag}(1, -1, -1)$. At $p = (x, y, t)$, $d\tau(X_p) = \partial_x - 2y \partial_t = X_{\tau(p)}$ and $d\tau(Y_p) = -\partial_y + 2x \partial_t = -Y_{\tau(p)}$, and $(\tau^*\theta)_p(v) = -v_t + 2(-xv_y + yv_x) = -\theta_p(v)$. So $d\tau$ maps H onto H , maps the orthonormal frame (X, Y) to the orthonormal frame $(X, -Y)$, and satisfies $d\tau \circ J = -J \circ d\tau$ on H . As in the proof of Proposition 5.1, τ preserves lengths of horizontal curves and is an involution, hence an isometry, so $H_\tau = 1$; it is anti-CR and, by Lemma 4.1(a), CR at no point. $\square$

The counterexample does not depend on the sign convention for J : a map is anti-CR for J if and only if it is anti-CR for $-J$. Both maps reverse the coorientation ($u = -1$), in accordance with Theorem 1.1(3), and preserve the orientation of the 3-manifold, in accordance with Remark 4.2. Not

every orthogonal map of $\mathbb{C}^2$ that reverses J in one variable gives an example: $(z, w) \mapsto (\bar{z}, w)$ maps S^3 to itself, but it does not map H into H (at $p = (\frac{3}{5}, 0, 0, \frac{4}{5})$ it maps $e_1(p) = (0, \frac{4}{5}, \frac{3}{5}, 0)$ to a vector on which θ takes the value $-\frac{24}{25}$), so by Theorem 1.1(1) it is not conformal for any Hermitian metrics.

6 Computations and reproducibility

No proof in this note depends on a computation. The identities of Section 5 and Remark 4.4 are short; they were checked symbolically (with SymPy) when the result was first obtained, and the program `check_note.py` of the reproducibility package re-checks them as identities of polynomials in exact rational arithmetic, with the Python standard library only, together with the CR structure induced from the Siegel domain, the sign rule of Theorem 1.1(3) for dilations, a rotation and their compositions with τ , and Lemma 4.1 for all 2400 nonzero integer 2×2 matrices with entries in $[-3, 3]$; all 33 checks pass in well under a second. The package also contains the SymPy program of the first computation and the programs of the verification runs described below with their recorded outputs, which were all reproduced when this note was written; the program of the last run, in the folder `independent_run_2`, performs 36 symbolic checks in about two seconds.

Verification. The result was first obtained with the proofs of Sections 3 to 5 and the SymPy check of Section 5. Two independent verification runs, both AI-assisted, followed; they examined the first write-up of the result. Each re-derived Lemmas 3.1, 3.2 and 4.1, the sign argument and Remark 4.2 line by line; checked the question against the LaTeX source of [BFGK95]; checked the definitions and the statement of [CCLO19, Theorem 1.1] and its applicability against the source, the first run in the accepted manuscript of the journal version and in arXiv v2, the second in arXiv v2; and recomputed the identities of Section 5 with its own SymPy program by a different route (the first run with the $(1, 0)$ -field $\bar{w} \partial_z - \bar{z} \partial_w$ of S^3). Both re-ran the original program and obtained identical output. The second run derived the CR structure of the Heisenberg group from the Siegel domain, tested the sign rule of Theorem 1.1(3) on the boundary values of a biholomorphic inversion of the Siegel domain and on its composition with τ , and, as an illustration only, estimated the sphere distortion numerically from a closed formula for the Heisenberg distance: exactly 1 for τ , tending to 1 for the inversion and its composition with τ , and 2 for the contact map $(2x, y, 2t)$, which is not conformal. Neither run found a mathematical error. Their required corrections concern the presentation and the bibliography, and all were made: Remark 2.3 on the two definitions of the distance, the coorientation argument of Lemma 2.1, the wording “modulo [CCLO19, Theorem 1.1]” in every statement of the result, the numbering of [CCLO19] in both versions, one sentence in the proof of Lemma 3.1, the four authors of [BFGK95], the data of [CL21], the citation of [FV16], and the description of the related work as far as the sources read support it.

A further independent verification run, also AI-assisted, then checked the text of this note line by line. It read the sources again: the LaTeX source of [BFGK95], which it compiled to obtain the numbering of the typeset list; the arXiv source of [Tan96] (Definition 2.1, Theorem 2.3 and formula (2.9)); the accepted manuscript and the arXiv v2 source of [CCLO19], compiled for its numbering (Section 2.1, Definitions 3.1, 3.2, 3.13, Theorem 3.3, Theorem 1.1, Section 5.1 and the citations of [Tan96]); the arXiv source of [CLOW20]; the zbMATH reviews of [KR90, FV16]; and, of [WW12], the copy of the Russian original fetched on 19 October (Math-Net.Ru refused the request of 20 October). It re-derived Lemmas 2.1, 3.1, 3.2 and 4.1, Steps 0–5 and the remarks, and wrote its own SymPy program by routes not used before: the CR structure of S^3 from its defining function, the pull-back of θ to Hopf coordinates, the CR structure of the Heisenberg group from the Siegel embedding, σ and τ as restrictions of antiholomorphic maps of $\mathbb{C}^2$, the sign of u for the inversion $(z, w) \mapsto (z/w, -1/w)$ of the Siegel domain ($u = 1/|w|^2$, CR) and for its composition with τ ($u =$

$-1/|w|^2$, anti-CR), the controls $(z, w) \mapsto (\bar{z}, w)$ (not contact) and $(x, y, t) \mapsto (2x, y, 2t)$ (contact, neither CR nor anti-CR), Remark 4.4, and Lemma 4.1 for all 6560 nonzero integer matrices with entries in $[-4, 4]$; all 36 checks pass, and a negative control makes 10 of them fail. It re-ran all programs of the package from the source archive and obtained identical outputs, and it repeated the searches described below. It found no mathematical error. Its required corrections were made: the numbering of the problem list, the arXiv numbering of Section 5.1 of [CCLO19], the numbers of [CLOW20], the regularity C^2 in Remark 4.2, the description of orientation in [WW12], the credit to [WF14], and the record of the searches. The proof of [CCLO19, Theorem 1.1] was checked by none of the runs; it is cited.

Scope and priority. The note answers Tang’s question for the Carnot–Carathéodory metric reading, as a corollary of [CCLO19, Theorem 1.1] and elementary linear algebra: no as literally posed, yes under the coorientation convention ($\lambda > 0$), in the precise form of Theorem 1.1. The analytic reading of [Tan96, Definition 2.1] remains open: Theorem 1.1 covers it only for maps that are in addition metrically quasiconformal (Remark 4.3). The higher-dimensional case and K -quasiconformal maps with $K > 1$ are not treated. The question is Tang’s [BFGK95]; the prior partial result is [Tan96, Theorem 2.3]; the whole analytic content is [CCLO19, Theorem 1.1]; the dichotomy with the same linear algebra is in [CLOW20] for the flat model (and in [WF14] for Goursat groups) and, almost everywhere on embedded hypersurfaces, in [WW12, Corollary 1.1]; for the related work see Section 1.3. The fitting label is “answered by combining published results”.

What was read: [BFGK95] in its LaTeX source (both copies of the question and the companion problem); [Tan96] in the arXiv version (the introduction, Definition 2.1, Theorem 2.3 and its proof); of [CCLO19], in the accepted manuscript and in arXiv v2, the introduction, Section 2.1, the definitions and statements of Section 3, Section 5.1 and part of Section 6, while the proof of Theorem 1.1 was not checked; [CLOW20] in the arXiv version v2 (Corollary 3.2, Theorem 3.3 and its proof); [WW12] in the Russian original from Math-Net.Ru, Section 1 and the definition of orientation after Proposition 3.1, not the proofs. Of [Tan94, WF14, CL21] only the abstracts were read, and of [KR90, FV16] only the zbMATH reviews; [KR85, KR95, Cho40] were not read, and what we say about them is taken from [Tan96], [WW12] and [CCLO19].

The searches were made on 19 and 20 October 2026: queries to the arXiv API (for 1-quasiconformal and conformal maps of CR manifolds, anti-CR maps, quasiconformal maps of pseudoconvex hypersurfaces, Liouville theorems on CR manifolds, and the authors), to zbMATH (the papers of Tang, Korányi and Reimann, anti-CR and 1-quasiconformal maps, and Liouville theorems on CR manifolds), to Crossref, the lists of citing works in Semantic Scholar (about 25 works citing [CCLO19] and 17 citing [Tan96], titles screened) and in OpenAlex (16 works citing [Tan96], and 20 and 8 works citing the two OpenAlex records of [CCLO19]; screened when the result was first obtained and again on 20 October), and five web searches. No printed answer to the question was found in these sources, and no paper stating the conclusion of Theorem 1.1 for general strongly pseudoconvex CR 3-manifolds or naming Tang’s question; a search that finds nothing is not a proof of novelty. No priority is claimed beyond the three points listed at the end of Section 1.3.

AI-assisted tools were used to search the problem corpus and the literature, to find the arguments, to write and run the verification programs, and to help prepare this manuscript. The author has reviewed the proofs, the computations and the sources and is responsible for all claims. The note has not been peer reviewed.

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