

A Nonremovable Meromorphic Map into a Ball on a Codimension-Two Wedge

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Abstract

We construct a meromorphic germ in $\mathbb{C}^3$ that is holomorphic from an attached wedge into the open unit ball, extends continuously to its real-algebraic generic edge with values on the unit sphere, and nevertheless has no holomorphic extension to a full neighborhood of the edge point. The edge has real codimension two and CR dimension one. The construction combines an explicit rational sphere identity with a holomorphic factor that is unimodular on the edge and suppresses interior norm growth. The boundary limit holds along every wedge approach, not just along normal rays. This gives a negative answer to the arbitrary-generic-edge question posed by Meylan in the 2010 AIM CR-mapping problem list, with convergence understood from inside the ball. The example is nonminimal and has a continuous but nondifferentiable boundary map; it makes no claim about strengthened minimal-source or smooth-boundary-map formulations. This manuscript is unrefereed.

1 The extension question and the example

Problem 2 in the section “Mappings between balls” of the 2010 AIM workshop list [1], attributed to F. Meylan, asks whether a meromorphic germ near a real-analytic generic submanifold must extend holomorphically to a full neighborhood when it is holomorphic in an attached one-sided wedge and its squared norm approaches one from below toward the edge. The list places no minimality, finite-type or differentiable-boundary-map condition on the source data. The source hypersurface case is due to Chiappari [2].

We use the following explicit boundary interpretation: the image of the wedge lies in the open unit ball, and the norm tends to one along every sequence approaching every nearby edge point. This is the from-inside meaning of “increases to 1” in [1]; no additional ordered family of paths or differential monotonicity condition is specified there. Our map has the stronger property of continuous vector-valued boundary limits. We do not assert monotonicity along arbitrary backtracking curves.

Use coordinates $(z, t, w) \in \mathbb{C}^3$, with $a = \operatorname{Re} t$, $v = \operatorname{Im} t$, $u = \operatorname{Re} w$ and $h = \operatorname{Im} w$. Define

$$M = \{\operatorname{Im} t = 0, \quad \operatorname{Im} w = |z|^2 + (\operatorname{Re} t)^2\}. \quad (1)$$

Let U_M be its open piece parameterized by $|z|, |a|, |u| < 1/8$, and set

$$W = \left\{ (z, a + iv, u + i(|z|^2 + a^2 + s)) : \begin{array}{l} |z|, |a|, |u| < 1/8, \\ 0 < v < s < 1/8 \end{array} \right\}. \quad (2)$$

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Thus W is obtained by adding a fixed open transverse cone to U_M , with a local truncation. In particular, it is a full-dimensional open wedge, not just a family of curves.

Put

$$q = -t^4 + iw, \quad F = \left(\frac{2t^2 z}{q}, \frac{2t^3}{q}, \frac{-t^4 - iw}{q} \right), \quad G = e^{100it} F. \quad (3)$$

All numerators of G are holomorphic and the exponential is a unit. The number 100 is a convenient bound, not an optimal constant.

Theorem 1.1. *The edge M is smooth, real-algebraic and generic, of real codimension two and CR dimension one. The meromorphic germ G is holomorphic on W , satisfies $G(W) \subset \mathbb{B}^3$, and extends continuously to $W \cup U_M$ with $G(U_M) \subset \partial\mathbb{B}^3$ and $G(0) = (0, 0, -1)$. Nevertheless, no holomorphic map on a full neighborhood of 0 agrees with G on the wedge. The edge is nonminimal, and the boundary map is not differentiable at 0.*

Theorem 1.1 also supplies examples with target $\mathbb{B}^N$ for any $N \geq 3$ by appending zero components. It concerns a local wedge, not a proper map on the whole boundary of a bounded domain.

2 Uniform estimates and the ball image

For the defining functions $\rho_1 = \operatorname{Im} t$ and $\rho_2 = \operatorname{Im} w - |z|^2 - (\operatorname{Re} t)^2$, the t, w minor of $(\partial\rho_j)$ is

$$\det \begin{pmatrix} 1/(2i) & 0 \\ -a & 1/(2i) \end{pmatrix} = -\frac{1}{4}.$$

This proves smoothness and genericity. The two imaginary coordinate directions used in (2) form a transverse complement. The leaves with $t = a$ fixed are proper CR submanifolds with CR dimension one, so M is nonminimal.

Every point of W has all three coordinate moduli less than $1/2$ and satisfies

$$v > 0, \quad h > |z|^2 + a^2 + v. \quad (4)$$

Indeed, $|t|^2 < 1/32$ and $|w|^2 < (1/8)^2 + (5/32)^2 < 1/4$. We work in the larger region described by (4) and these $1/2$ coordinate bounds.

Fix such a point and consider $t_\tau = a + i\tau$, $0 \leq \tau \leq v$, with z, w fixed. The segment is auxiliary; its real- t endpoint need not lie in the open wedge. Along the entire segment,

$$|z|^2 + |t_\tau|^2 \leq |z|^2 + a^2 + v^2 < h, \quad |t_\tau|^4 \leq \frac{1}{4}|t_\tau|^2 \leq h/4. \quad (5)$$

Because $\operatorname{Re} q = -\operatorname{Re}(t_\tau^4) - h \leq -3h/4$,

$$|q| \geq 3h/4 > 0. \quad (6)$$

In particular, F and G are holomorphic on W .

From (5)–(6) and $F_3 = -1 - 2t^4/q$,

$$|F_1| \leq \frac{8}{3}|z|, \quad |F_2| \leq \frac{8}{3}|t_\tau|, \quad |F_3 + 1| \leq \frac{8}{3}|t_\tau|^2. \quad (7)$$

Hence $\|F\|^2 \leq 19/3 < 9$. Differentiation with z, w fixed gives

$$\begin{aligned} \partial_t F_1 &= 4tz/q + 8t^5 z/q^2, \\ \partial_t F_2 &= 6t^2/q + 8t^6/q^2, \\ \partial_t F_3 &= -8t^3/q - 8t^7/q^2. \end{aligned} \quad (8)$$

Use $|t||z| \leq h/2$, $|t|^2/h \leq 1$, $|t|^4/h \leq 1/4$ and $|t| \leq 1/2$. The three absolute values in (8) are bounded respectively by

$$\frac{8}{3} + \frac{16}{9} = \frac{40}{9} < 12, \quad 8 + \frac{32}{9} = \frac{104}{9} < 12, \quad \frac{16}{3} + \frac{16}{9} = \frac{64}{9} < 12.$$

Thus $\|\partial_t F\| \leq 24$ and

$$\left| \frac{d}{d\tau} \|F(z, a + i\tau, w)\|^2 \right| \leq 2\|F\| \|\partial_t F\| \leq 144. \quad (9)$$

For real $t = a$, a direct polynomial identity is

$$|q|^2 - |2a^2 z|^2 - |2a^3|^2 - |-a^4 - iw|^2 = 4a^4(h - |z|^2 - a^2). \quad (10)$$

In particular, the starting norm $\|F(z, a, w)\|^2$ is at most one. Integration of (9) gives

$$\|F(z, a + iv, w)\|^2 \leq 1 + 144v.$$

Since $|e^{100it}|^2 = e^{-200v}$ and $v > 0$,

$$\|G\|^2 \leq e^{-200v}(1 + 144v) < 1. \quad (11)$$

The strict inequality follows from $e^{200v} \geq 1 + 200v > 1 + 144v$. It proves ball inclusion throughout W without any numerical approximation of the exponential.

The damping cannot simply be discarded. At $z = 0$, $t = (1 + i)/20$ and $w = 41i/400$, which is a point of W , the last component of F alone has modulus greater than one. The uniform estimate (11) repairs this interior defect while leaving the boundary norm unchanged.

3 Boundary limits and the nonremovable pole

On M , $t = a$ is real and the exponential has modulus one. If $q = 0$ there, then $w = -ia^4$, so

$$|z|^2 + a^2 + a^4 = 0.$$

Thus q vanishes on M only at 0. At every other nearby edge point G is holomorphic in a full neighborhood and has norm one by (10).

At the origin, (7) gives, uniformly throughout W ,

$$F_1 \longrightarrow 0, \quad F_2 \longrightarrow 0, \quad F_3 \longrightarrow -1.$$

The factor e^{100it} tends to one. These limits hold for every wedge approach, including tangential approaches. On $M \setminus \{0\}$ the same estimates apply when $h = |z|^2 + a^2 > 0$; if $h = 0$ and $w \neq 0$ is real, F is exactly $(0, 0, -1)$. Therefore G extends continuously to $W \cup U_M$, with the claimed sphere-valued trace.

Suppose now that a holomorphic map H on a full connected neighborhood V of 0 agreed with G on $V \cap W$. The holomorphic identity

$$qH_2 = 2t^3 e^{100it} \quad (12)$$

holds on a nonempty open set, hence on V . Restriction to the smooth complex hypersurface $w = -it^4$ makes the left side zero, while the right side is nonzero for small nonzero t . This

contradiction proves nonextension. Equivalently, the denominator is irreducible and does not divide the second numerator.

A separate check makes the off-wedge obstruction explicit. At $z = 0$, $t = \epsilon > 0$ and $w = -i\epsilon^4 + \epsilon^5$,

$$|G_2|^2 = 4/\epsilon^4 \longrightarrow \infty.$$

These points lie outside the wedge, so this does not contradict (11).

Finally, the boundary map is not differentiable at 0. For fixed $c \in \mathbb{R}$, the edge curves $z = 0$, $t = a$, $w = (c + i)a^2$ have the same tangent at 0, but

$$\frac{G_2}{a} = \frac{2e^{100ia}}{-(1 + a^2) + ic} \longrightarrow \frac{2}{-1 + ic}.$$

The limits differ, for example for $c = 0$ and $c = 1$. This finishes the proof of Theorem 1.1.

4 Scope, antecedents and reproducibility

Chiappari’s extension theorem [2] has a hypersurface source; Yazici’s generalization [4] also retains that hypothesis. Mir–Zaitsev [3, Theorem 1.4] require a minimal generic source and smooth CR map germs, obtain meromorphic continuation, and state holomorphic continuation in the hypersurface case. Our source has codimension two, is nonminimal, and has a nondifferentiable boundary map. The example neither contradicts those results nor answers the strengthened minimal-source question.

The frozen UnsolvedMath record [5] contains a polar-edge uniqueness criterion and identifies the obstruction of a polar hypersurface avoiding the wedge while meeting its edge in too small a set to force cancellation. We credit that viewpoint. Its bidisc rational-inner example fails the all-approaches boundary condition. The present construction realizes the obstruction with a single continuous sphere-valued trace. The quotient and damping ingredients are elementary; we make no absolute-priority claim from a bounded literature search.

The accompanying standard-library Python checker verifies the sphere identity and quotient derivatives coefficientwise, and checks 21,397 exact rational conditions, including 59 edge points, 360 wedge points, 1,800 segment points and six negative controls. Normal and optimized runs agree. These finite tests support, but do not replace, the uniform analytic proof above. No formal proof-assistant verification or independent review is claimed.

AI assistance was used in developing the construction, searching related literature, performing the detailed self-audit, preparing the exact checker and drafting this manuscript. The work is an unrefereed preprint; the explicit theorem and its stated limitations govern its claims.

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