

IPhO 1983

Theory problems and solutions · Bucharest, Romania · 5 theory problems with complete solutions

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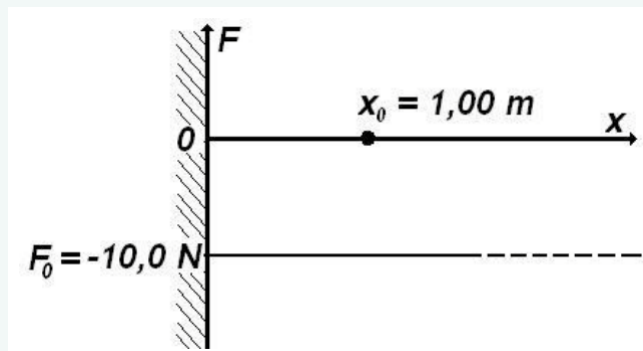
Problem 1: Jumping particle

A particle moves along the positive axis Ox (one-dimensional situation) under a force that's projection on Ox is $F_x = F_0$ as represented in the figure below (as function of x). At the origin of Ox axis is placed a perfectly reflecting wall.

A friction force of constant modulus $F_f = 1,00\text{ N}$ acts anywhere the particle is situated.

The particle starts from the point $x = x_0 = 1,00\text{ m}$ having the kinetic energy $E_c = 10,0\text{ J}$.

- Find the length of the path of the particle before it comes to a final stop
- Sketch the potential energy $U(x)$ of the particle in the force field F_x .
- Draw qualitatively the dependence of the particle speed as function of his coordinate x .



Official English text of the problem.

Answers

- a.** The total path length is $L = 20,0\text{ m}$. The final resting position is the wall, $x = 0$.
- b.** Choosing $U(0) = 0$,

$$U(x) = (10,0\text{ N})x, \quad x \geq 0.$$

Its graph is a straight line through the origin with slope $10,0\text{ J m}^{-1}$.

- **c.** The speed–position graph consists of square-root branches representing successive outward and inward passages. Each successive outer turning distance is $9/11$ times the preceding one; successive wall speeds decrease by $\sqrt{9/11}$. The branches converge to $(x, v) = (0, 0)$. For initial motion toward $+x$, the outer turning points are

$$a_n = \frac{21}{11} \left(\frac{9}{11} \right)^n \text{ m}, \quad n = 0, 1, 2, \dots$$

If the initial motion is toward the wall, replace $21/11$ by $19/11$, after the initial inward passage.

Solution

Write the field force as $F_x = -F$, where

$$F = 10.0 \text{ N}, \quad f = F_f = 1.00 \text{ N}.$$

Friction opposes the motion. Consequently, the net force is $-(F + f)$ during outward motion and $-(F - f)$ during inward motion. A perfectly reflecting wall reverses the velocity without changing the kinetic energy.

a. Total path length

Since $F > f$, the particle cannot remain at rest at any $x > 0$: whenever it reaches an outer turning point, it starts moving back toward the wall.

To establish the eventual resting position, consider a turning point at distance A from the wall. During the inward trip, the field does work FA and friction does work $-fA$. Thus the kinetic energy on reaching the wall is

$$K_w = (F - f)A.$$

After reflection, this energy is spent moving outward against both the field and friction. The next turning distance A' therefore satisfies

$$(F + f)A' = (F - f)A,$$

so

$$\frac{A'}{A} = \frac{F - f}{F + f} = \frac{9}{11} < 1.$$

The turning distances and wall speeds tend to zero. The final resting position is therefore $x_f = 0$. Moreover, each half-trip takes a time proportional to the square root of its turning distance; the remaining times form convergent geometric sums. Thus, in this idealized model, infinitely many reflections accumulate in a finite time.

Now apply the work–energy theorem to the entire motion. The total work of the constant field depends only on the endpoints:

$$W_x = F_x(x_f - x_0) = (-10.0 \text{ N})(-1.00 \text{ m}) = 10.0 \text{ J}.$$

Friction does work $-fL$, while the elastic reflections cause no kinetic-energy loss. Hence

$$0 - E_c = W_x - fL.$$

Therefore

$$L = \frac{E_c + Fx_0}{f} = \frac{10.0 \text{ J} + 10.0 \text{ J}}{1.00 \text{ N}} = 20.0 \text{ m}.$$

This result is independent of the initial direction of motion.

b. Potential energy

The potential energy associated with the field obeys

$$F_x = -\frac{dU}{dx}.$$

Since $F_x = -F$,

$$\frac{dU}{dx} = F, \quad U(x) = Fx + C.$$

With $U(0) = 0$,

$$U(x) = (10.0 \text{ N})x, \quad x \geq 0.$$

The sketch is a straight, rising line through $(0, 0)$, passing through

$$(x, U) = (1.00 \text{ m}, 10.0 \text{ J})$$

and

$$(x, U) = (2.00 \text{ m}, 20.0 \text{ J}).$$

The wall restricts the accessible region to $x \geq 0$. Friction is dissipative and is not included in this potential.

c. Speed as a function of position

Let the particle mass be m , and define its initial speed by

$$v_0 = \sqrt{\frac{2E_c}{m}}.$$

Since no mass is supplied, v_0 provides a convenient vertical scale.

First take the initial velocity to point toward $+x$. On the initial outward passage, work–energy gives

$$\frac{1}{2}mv^2 = E_c - (F + f)(x - x_0).$$

The first turning point is consequently

$$a_0 = x_0 + \frac{E_c}{F + f} = 1.00 \text{ m} + \frac{10.0 \text{ J}}{11.0 \text{ N}} = \frac{21}{11} \text{ m} \approx 1.91 \text{ m}.$$

Using the ratio found in part a, all subsequent turning points are

$$a_n = a_0 r^n, \quad r = \frac{9}{11}.$$

For an outward passage ending at a_n , the remaining kinetic energy equals the work needed to reach that turning point:

$$v_{\rightarrow, n}(x) = \sqrt{\frac{2(F + f)}{m}(a_n - x)}.$$

For $n = 0$, only $x_0 \leq x \leq a_0$ is traversed. For $n \geq 1$, the full interval $0 \leq x \leq a_n$ is traversed.

On the inward passage from a_n , the net force toward the wall has magnitude $F - f$, giving

$$v_{\leftarrow,n}(x) = \sqrt{\frac{2(F-f)}{m}(a_n - x)}, \quad 0 \leq x \leq a_n.$$

Both expressions denote **speed**, so both are nonnegative.

At the wall, the inward branch from a_n joins the next outward branch continuously in speed:

$$v_{w,n} = \sqrt{\frac{2(F-f)a_n}{m}} = \sqrt{\frac{2(F+f)a_{n+1}}{m}}.$$

Thus

$$\frac{v_{w,n+1}}{v_{w,n}} = \sqrt{\frac{9}{11}}.$$

The qualitative sketch can be constructed as follows:

- Start at $(x, v) = (1.00 \text{ m}, v_0)$. Follow a decreasing, concave-down square-root curve to $(1.91 \text{ m}, 0)$.
- Return along the inward branch, moving left and upward on the graph, to

$$(x, v) = \left(0, \sqrt{\frac{189}{110}} v_0\right) \approx (0, 1.31 v_0).$$

- After reflection, follow the next outward branch to

$$(x, v) = (a_1, 0), \quad a_1 = \frac{189}{121} \text{ m} \approx 1.56 \text{ m}.$$

- Repeat with successively smaller turning distances. Every branch has a vertical tangent at its outer turning point. For the same turning distance, the outward branch lies above the inward branch.
- The sequence converges to the final point $(0, 0)$.

There are generally several speeds at the same position, corresponding to different passages; the complete graph is therefore not a single-valued function of x .

If the initial velocity instead points **toward the wall**, the initial branch is

$$v(x) = \sqrt{\frac{2}{m}[E_c + (F-f)(x_0 - x)]}, \quad 0 \leq x \leq x_0.$$

The first wall encounter has kinetic energy

$$E_c + (F-f)x_0 = 19.0 \text{ J},$$

so its speed is $\sqrt{1.90} v_0$. After this reflection, the turning points are

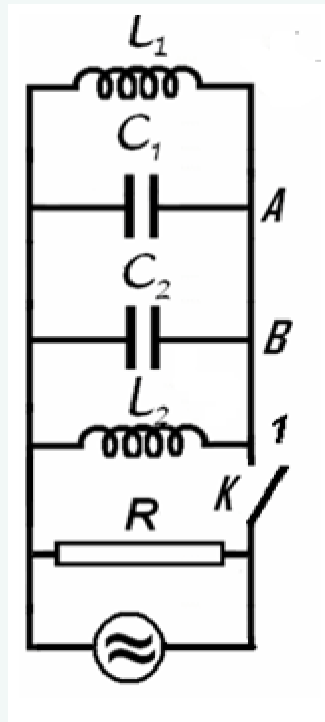
$$b_n = \frac{19}{11} \left(\frac{9}{11}\right)^n \text{ m}.$$

The same outward and inward branch formulas apply with a_n replaced by b_n .

The solution was checked against the official solution.

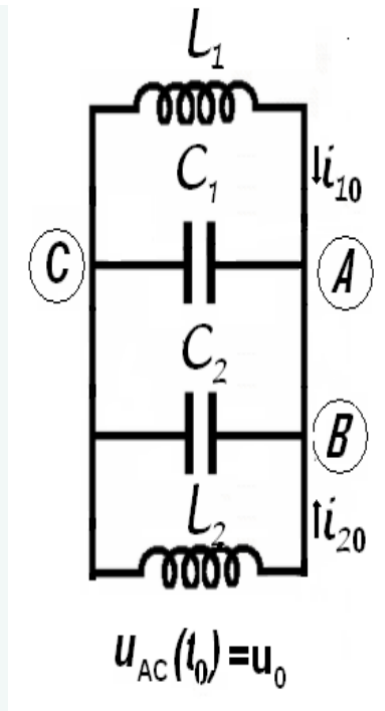
Problem 2: Different kind of oscillation

Let's consider the electric circuit in the figure, for which $L_1 = 10 \text{ mH}$, $L_2 = 20 \text{ mH}$, $C_1 = 10 \text{ nF}$, $C_2 = 5 \text{ nF}$ and $R = 100 \text{ k}\Omega$. The switch K being closed the circuit is coupled with a source of alternating current. The current furnished by the source has constant intensity while the frequency of the current may be varied.



a. Find the ratio of frequency f_m for which the active power in circuit has the maximum value P_m and the frequency difference $\Delta f = f_+ - f_-$ of the frequencies f_+ and f_- for which the active power in the circuit is half of the maximum power P_m .

The switch K is now open. In the moment t_0 immediately after the switch is open the intensities of the currents in the coils L_1 and L_2 are $i_{01} = 0,1 \text{ A}$ and $i_{02} = 0,2 \text{ A}$ (the currents flow as in the figure); at the same moment, the potential difference on the capacitor with capacity C_1 is $u_0 = 40 \text{ V}$:



- b.** Calculate the frequency of electromagnetic oscillation in $L_1C_1C_2L_2$ circuit;
- c.** Determine the intensity of the electric current in the AB conductor;
- d.** Calculate the amplitude of the oscillation of the intensity of electric current in the coil L_1 .

Neglect the mutual induction of the coils, and the electric resistance of the conductors. Neglect the fast transition phenomena occurring when the switch is closed or opened.

Official English text of the problem.

Answers

- a.** $\frac{f_m}{\Delta f} = 150$, with $f_m = 15.9 \text{ kHz}$ and $\Delta f = 106 \text{ Hz}$.
- b.** $f_0 = 15.9 \text{ kHz}$, corresponding to $\omega_0 = 1.00 \times 10^5 \text{ rad s}^{-1}$.
- c.** $I_{AB} = 0.100 \text{ A}$, constant and directed **from B to A**.
- d.** $I_{1,\text{osc}} = 0.204 \text{ A}$. The current oscillates about a mean of -0.100 A , taking the indicated initial-current direction as positive.

Solution

a. Driven circuit: resonance and half-power bandwidth

With K closed, both capacitors, both inductors, and the resistor are connected in parallel. Their equivalent capacitance and inductance are

$$C = C_1 + C_2 = 15 \text{ nF}, \quad L = \left(\frac{1}{L_1} + \frac{1}{L_2} \right)^{-1} = \frac{L_1 L_2}{L_1 + L_2} = 6.6667 \text{ mH}.$$

The inductance formula uses the stipulated absence of mutual induction.

Let I be the constant rms source current and $\omega = 2\pi f$. The circuit admittance is

$$Y = \frac{1}{R} + i \left(\omega C - \frac{1}{\omega L} \right).$$

Only R dissipates active power. Since the rms voltage is $I/|Y|$,

$$P(\omega) = \frac{I^2}{R|Y|^2} = \frac{I^2 R}{1 + R^2 \left(\omega C - \frac{1}{\omega L} \right)^2}.$$

The maximum occurs when the reactive part of the admittance vanishes:

$$\omega_m C - \frac{1}{\omega_m L} = 0.$$

Consequently,

$$\omega_m = \frac{1}{\sqrt{LC}}, \quad P_m = I^2 R.$$

At either half-power frequency,

$$P = \frac{P_m}{2} \implies \left| \omega C - \frac{1}{\omega L} \right| = \frac{1}{R}.$$

The two positive solutions are

$$\omega_{\pm} = \sqrt{\omega_m^2 + \frac{1}{4R^2 C^2}} \pm \frac{1}{2RC}.$$

Their difference is therefore exactly

$$\Delta\omega = \frac{1}{RC}, \quad \Delta f = \frac{1}{2\pi RC}.$$

No narrow-band approximation is needed.

Numerically,

$$LC = 1.00 \times 10^{-10} \text{ s}^2,$$

so

$$f_m = \frac{10^5}{2\pi} = 1.59155 \times 10^4 \text{ Hz},$$

and

$$\Delta f = \frac{1}{2\pi(10^5)(15 \times 10^{-9})} = 106.103 \text{ Hz}.$$

Thus the requested ratio is

$$\boxed{\frac{f_m}{\Delta f} = \omega_m RC = R \sqrt{\frac{C}{L}} = 150}.$$

b. Free electromagnetic oscillation

After the switch opens, the resistor and source no longer load the LC circuit. Under the stated idealizations, its oscillation is undamped.

Define

$$u(t) = V_A - V_C = V_B - V_C,$$

and take both inductor currents i_1, i_2 as positive from the left rail to the right rail. These agree with the indicated initial directions, so

$$i_1(t_0) = 0.1 \text{ A}, \quad i_2(t_0) = 0.2 \text{ A}, \quad u(t_0) = 40 \text{ V}.$$

The inductor voltage laws give

$$L_1 \dot{i}_1 = -u, \quad L_2 \dot{i}_2 = -u.$$

The total capacitor current from right to left is $C\dot{u}$. Kirchhoff's current law therefore gives

$$C\dot{u} = i_1 + i_2.$$

Differentiating,

$$C\ddot{u} = \dot{i}_1 + \dot{i}_2 = -\left(\frac{1}{L_1} + \frac{1}{L_2}\right)u,$$

or

$$\ddot{u} + \omega_0^2 u = 0, \quad \omega_0^2 = \frac{1}{C} \left(\frac{1}{L_1} + \frac{1}{L_2} \right) = \frac{1}{LC}.$$

Hence

$$\boxed{\omega_0 = 1.00 \times 10^5 \text{ rad s}^{-1}}, \quad \boxed{f_0 = \frac{\omega_0}{2\pi} = 15.9 \text{ kHz}}.$$

For the remaining parts, the initial voltage derivative is

$$\dot{u}(t_0) = \frac{i_{01} + i_{02}}{C} = \frac{0.3}{15 \times 10^{-9}} = 2.00 \times 10^7 \text{ V s}^{-1}.$$

Writing $\theta = \omega_0(t - t_0)$, the voltage is therefore

$$\boxed{u(t) = (40 \cos \theta + 200 \sin \theta) \text{ V}}.$$

c. Current in conductor AB

Let j be the current in AB , positive from A to B . At A , the incoming inductor current splits between C_1 and AB :

$$j = i_1 - C_1 \dot{u}.$$

Its derivative is

$$\dot{j} = -\frac{u}{L_1} - C_1 \ddot{u} = \left(C_1 \omega_0^2 - \frac{1}{L_1} \right) u.$$

For the given components,

$$L_1 C_1 = L_2 C_2 = 1.00 \times 10^{-10} \text{ s}^2.$$

This implies

$$\omega_0^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2},$$

and consequently

$$\dot{j} = 0.$$

Thus the current in AB is constant despite the oscillating capacitor voltage and coil currents.

Its value follows from the initial conditions:

$$j = i_{01} - C_1 \frac{i_{01} + i_{02}}{C_1 + C_2} = 0.1 - \frac{10}{15}(0.3) = -0.100 \text{ A}.$$

The negative sign reverses the chosen reference direction:

$$I_{AB} = 0.100 \text{ A, from } B \text{ to } A.$$

d. Oscillation amplitude of the current in L_1

Using $i_1 = j + C_1 \dot{u}$ and the results above,

$$\begin{aligned} i_1(t) &= -0.100 + C_1 \omega_0 (-40 \sin \theta + 200 \cos \theta) \\ &= (-0.100 + 0.200 \cos \theta - 0.0400 \sin \theta) \text{ A}. \end{aligned}$$

For an oscillatory term $a \cos \theta + b \sin \theta$, the amplitude is $\sqrt{a^2 + b^2}$. Therefore,

$$I_{1,\text{osc}} = \sqrt{(0.200)^2 + (0.0400)^2} = 0.20396 \text{ A} \simeq 0.204 \text{ A}.$$

The constant term represents a persistent circulating current supported by the ideal inductors. Thus 0.204 A is the amplitude about the mean -0.100 A; the signed current ranges from approximately -0.304 A to $+0.104$ A.

The solution was checked against the official solution. Where this solution differs from the official solution, the official solution contains an error or a misprint.

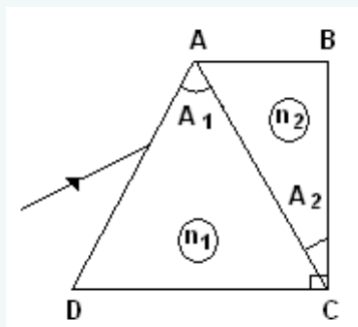
Problem 3: Prisms

Two dispersive prisms having apex angles $\hat{A}_1 = 60^\circ$ and $\hat{A}_2 = 30^\circ$ are glued as in the figure below ($\hat{C} = 90^\circ$). The dependences of refraction indexes of the prisms on the wavelength are given by the relations

$$\begin{aligned} n_1(\lambda) &= a_1 + \frac{b_1}{\lambda^2}; \\ n_2(\lambda) &= a_2 + \frac{b_2}{\lambda^2} \end{aligned}$$

were

$$a_1 = 1,1, \quad b_1 = 1 \cdot 10^5 \text{ nm}^2, \quad a_2 = 1,3, \quad b_2 = 5 \cdot 10^4 \text{ nm}^2.$$



- a. Determine the wavelength λ_0 of the incident radiation that pass through the prisms without refraction on AC face at any incident angle; determine the corresponding refraction indexes of the prisms.
- b. Draw the ray path in the system of prisms for three different radiations λ_{red} , λ_0 , λ_{violet} incident on the system at the same angle.
- c. Determine the minimum deviation angle in the system for a ray having the wavelength λ_0 .
- d. Calculate the wavelength of the ray that penetrates and exits the system along directions parallel to DC .

Official English text of the problem.

Answers

- a. $\lambda_0 = 500 \text{ nm}$, with $n_1(\lambda_0) = n_2(\lambda_0) = 1.5$.
- b. At AC , red bends toward the normal, λ_0 continues straight, and violet bends away from the normal. For a common horizontal incident beam, the emerging rays are ordered **red**, λ_0 , **violet** from top to bottom, with increasing downward deviation.
- c. $\delta_{\min} = 2 \arcsin(1.5 \sin 15^\circ) - 30^\circ \approx 15.69^\circ$.
- d. $\lambda \approx 1.18 \times 10^3 \text{ nm} = 1.18 \mu\text{m}$, in the infrared.

Solution

We use geometrical optics and Snell's law, taking the refractive index of the surrounding air as 1 and neglecting any optical effect of the glue. No small-angle approximation is used.

The geometry gives

$$\angle ACD = 90^\circ - A_2 = 60^\circ, \quad \angle ADC = 180^\circ - A_1 - \angle ACD = 60^\circ.$$

Consequently, measured relative to the rightward direction parallel to DC :

- the inward normal to AD points 30° downward;
- the normal to AC , directed from prism 1 into prism 2, points 30° upward;
- the outward normal to BC is horizontal.

a. Wavelength with no refraction at AC

At the common face, Snell's law is

$$n_1(\lambda) \sin i = n_2(\lambda) \sin t.$$

For the direction to remain unchanged, $t = i$. Requiring this for every incidence angle, rather than just normal incidence, gives

$$n_1(\lambda_0) = n_2(\lambda_0).$$

Thus

$$a_1 + \frac{b_1}{\lambda_0^2} = a_2 + \frac{b_2}{\lambda_0^2},$$

and therefore

$$\lambda_0^2 = \frac{b_1 - b_2}{a_2 - a_1} = \frac{5 \times 10^4 \text{ nm}^2}{0.2} = 2.5 \times 10^5 \text{ nm}^2.$$

Taking the positive wavelength,

$$\boxed{\lambda_0 = 500 \text{ nm}}.$$

The corresponding indices are

$$n_1(\lambda_0) = 1.1 + \frac{10^5}{500^2} = 1.5, \quad n_2(\lambda_0) = 1.3 + \frac{5 \times 10^4}{500^2} = 1.5.$$

b. Ray paths

Both indices decrease with increasing wavelength. Hence, at the entrance face AD , violet bends toward the normal more strongly than λ_0 , while red bends less strongly.

At the common face, the relevant index difference is

$$n_1(\lambda) - n_2(\lambda) = -0.2 + \frac{5 \times 10^4 \text{ nm}^2}{\lambda^2}.$$

Since

$$\lambda_{\text{violet}} < 500 \text{ nm} < \lambda_{\text{red}},$$

the refractions at AC are:

Radiation	Index comparison	Behaviour at AC
Red	$n_1 < n_2$	Bends toward the normal
λ_0	$n_1 = n_2$	Continues straight
Violet	$n_1 > n_2$	Bends away from the normal

A precise verbal sketch can use a common horizontal incident beam, whose incidence angle at AD is 30° . Choose its entry point sufficiently high that all three rays reach BC .

1. At AD , all three rays bend downward toward the inward normal. Inside prism 1, red is the upper ray, λ_0 the middle ray, and violet the lower ray.
2. At AC , draw its normal pointing 30° above horizontal. Red turns upward toward this normal, λ_0 remains straight, and violet turns farther downward away from the normal.
3. At BC , the normal is horizontal. The visible rays bend farther downward as they pass into air, away from this normal.

The emerging order is therefore **red above λ_0 , with violet below**, and violet has the greatest total deviation.

c. Minimum deviation for λ_0

For this wavelength, both prisms have index $n = 1.5$, so the internal face AC has no optical effect. The entrance and exit faces AD and BC act as the faces of a single prism with effective apex angle

$$\alpha = A_1 - A_2 = 30^\circ.$$

Let i, e be the external incidence and emergence angles, and r, r' the corresponding internal angles. Prism geometry and Snell's law give

$$\begin{aligned} r + r' &= \alpha, & \delta &= i + e - \alpha, \\ \sin i &= n \sin r, & \sin e &= n \sin r'. \end{aligned}$$

Therefore

$$\delta(r) = \arcsin(n \sin r) + \arcsin(n \sin(\alpha - r)) - \alpha.$$

The minimum occurs for the symmetric path,

$$r = r' = \frac{\alpha}{2}, \quad i = e.$$

For example, differentiating the preceding expression gives

$$\frac{d\delta}{dr} = \frac{n \cos r}{\sqrt{1 - n^2 \sin^2 r}} - \frac{n \cos(\alpha - r)}{\sqrt{1 - n^2 \sin^2(\alpha - r)}}.$$

This vanishes at $r = \alpha/2$ and changes from negative to positive there.

Thus

$$i_{\min} = \arcsin\left(n \sin \frac{\alpha}{2}\right) = \arcsin(1.5 \sin 15^\circ) \approx 22.8443^\circ,$$

and

$$\boxed{\delta_{\min} = 2i_{\min} - \alpha \approx 15.6886^\circ \approx 15.69^\circ}.$$

d. Parallel incident and emerging rays

The incident ray is horizontal, so its incidence angle at AD is 30° . If its refracted angle there is r , Snell's law gives

$$n_1 \sin r = \sin 30^\circ = \frac{1}{2}. \quad (1)$$

The emerging ray must also be horizontal. Since BC is vertical, emergence is normal to BC . Snell's law then requires the ray inside prism 2 to be horizontal as well.

The ray in prism 1 travels at angle $r - 30^\circ$ to the horizontal. Its incidence angle at AC is consequently

$$i_{AC} = 30^\circ - (r - 30^\circ) = 60^\circ - r.$$

The horizontal ray in prism 2 makes 30° with the normal to AC . Hence

$$n_1 \sin(60^\circ - r) = n_2 \sin 30^\circ = \frac{n_2}{2}. \quad (2)$$

Expanding equation (2) and using equation (1),

$$n_2 = \sqrt{3} n_1 \cos r - n_1 \sin r = \sqrt{3} \sqrt{n_1^2 - \frac{1}{4}} - \frac{1}{2}.$$

Squaring and simplifying gives

$$\boxed{3n_1^2 = n_2^2 + n_2 + 1}. \quad (3)$$

Introduce the positive, dimensionless quantity

$$u = \frac{5 \times 10^4 \text{ nm}^2}{\lambda^2}.$$

Then

$$n_1 = 1.1 + 2u, \quad n_2 = 1.3 + u.$$

Substitution into equation (3) yields

$$3(1.1 + 2u)^2 = (1.3 + u)^2 + (1.3 + u) + 1,$$

or

$$11u^2 + 9.6u - 0.36 = 0.$$

Only the positive root is physical:

$$u = \frac{-9.6 + \sqrt{108}}{22} \approx 0.0360139.$$

Therefore

$$\lambda = \sqrt{\frac{5 \times 10^4 \text{ nm}^2}{u}} \approx 1178.28 \text{ nm},$$

so

$$\boxed{\lambda \approx 1.18 \times 10^3 \text{ nm} = 1.18 \mu\text{m}}.$$

As a check, this wavelength gives

$$n_1 \approx 1.17203, \quad n_2 \approx 1.33601,$$

and $r \approx 25.2527^\circ$. The incidence angle at AC is then 34.7473° , which refracts to exactly 30° ; the ray subsequently reaches BC normally and exits horizontally.

The solution was checked against the official solution. Where this solution differs from the official solution, the official solution contains an error or a misprint.

Problem 4: Compton scattering

A photon of wavelength λ_i is scattered by a moving, free electron. As a result the electron stops and the resulting photon of wavelength λ_0 scattered at an angle $\theta = 60^\circ$ with respect to the direction of the incident photon, is again scattered by a second free electron at rest. In this second scattering process a photon with wavelength of $\lambda_f = 1,25 \times 10^{-10} \text{ m}$ emerges at an angle $\theta = 60^\circ$ with respect to the direction of the photon of wavelength λ_0 . Find the de Broglie wavelength for the first electron before the interaction. The following constants are known:

$h = 6,6 \times 10^{-34} \text{ J} \cdot \text{s}$ - Planck's constant

$m = 9,1 \times 10^{-31} \text{ kg}$ - mass of the electron

$c = 3,0 \times 10^8 \text{ m/s}$ - speed of light in vacuum

The purpose of the problem is to calculate the values of the speed, momentum and wavelength of the first electron.

To characterize the photons the following notation are used:

Table 4.1

	initial photon	photon – after the first scattering	final photon
momentum	$\vec{p}_i$	$\vec{p}_0$	$\vec{p}_f$

	initial photon	photon – after the first scattering	final photon
energy	E_i	E_0	E_f
wavelength	λ_i	λ_i	λ_f

To characterize the electrons one uses

Table 4.2

	first electron before collision	first electron after collision	second electron before collision	Second electron after collision
momentum	$\vec{p}_{1e}$	0	0	$\vec{p}_{2e}$
energy	E_{1e}	E_{0e}	E_{0e}	E_{2e}
speed	$\vec{v}_{1e}$	0	0	$\vec{v}_{2e}$

Official English text of the problem.

Answers

- **Problem IV (7 points)** — The first electron's initial speed, momentum magnitude, and de Broglie wavelength are

$$v_{1e} \approx 5.8 \times 10^6 \text{ m s}^{-1},$$

$$p_{1e} \approx 5.3 \times 10^{-24} \text{ kg m s}^{-1},$$

$$\lambda_{dB} \approx 1.24 \times 10^{-10} \text{ m}.$$

The intermediate photon wavelength is $\lambda_0 \approx 1.238 \times 10^{-10} \text{ m}$, and
 $\lambda_i = \lambda_f = 1.25 \times 10^{-10} \text{ m}$.

Solution

Problem IV. There are no numbered subparts in the supplied statement.

We apply conservation of energy and momentum to each collision. For a photon,

$$E = \frac{hc}{\lambda}, \quad p = \frac{E}{c} = \frac{h}{\lambda},$$

while an electron satisfies

$$E_e^2 = m^2 c^4 + p_e^2 c^2.$$

We initially use exact relativistic relations.

The second scattering determines λ_0 .

The second electron is initially at rest, so

$$E_0 + mc^2 = E_f + E_{2e}, \quad \vec{p}_0 = \vec{p}_f + \vec{p}_{2e}.$$

The angle between the incoming and outgoing photons is θ , giving

$$p_{2e}^2 c^2 = E_0^2 + E_f^2 - 2E_0 E_f \cos \theta.$$

Substitution into the electron's energy–momentum relation yields

$$(mc^2 + E_0 - E_f)^2 = m^2c^4 + E_0^2 + E_f^2 - 2E_0E_f \cos \theta.$$

After cancellation,

$$mc^2(E_0 - E_f) = E_0E_f(1 - \cos \theta).$$

Dividing by E_0E_f and substituting $E = hc/\lambda$, we obtain the Compton shift:

$$\lambda_f - \lambda_0 = \frac{h}{mc}(1 - \cos \theta).$$

With the given constants,

$$\frac{h}{mc} = \frac{6.6 \times 10^{-34}}{(9.1 \times 10^{-31})(3.0 \times 10^8)} = 2.41758 \times 10^{-12} \text{ m}.$$

Since $\cos 60^\circ = \frac{1}{2}$,

$$\Delta\lambda = \frac{h}{2mc} = 1.20879 \times 10^{-12} \text{ m},$$

and therefore

$$\lambda_0 = 1.25 \times 10^{-10} - 1.20879 \times 10^{-12} = 1.2379121 \times 10^{-10} \text{ m}.$$

The first scattering determines λ_i .

The first electron comes to rest after the collision. Thus

$$E_{1e} + E_i = mc^2 + E_0, \quad \vec{p}_{1e} + \vec{p}_i = \vec{p}_0.$$

Consequently,

$$E_{1e} = mc^2 + E_0 - E_i$$

and

$$p_{1e}^2 c^2 = E_0^2 + E_i^2 - 2E_0E_i \cos \theta.$$

Using the electron's energy–momentum relation again,

$$(mc^2 + E_0 - E_i)^2 = m^2c^4 + E_0^2 + E_i^2 - 2E_0E_i \cos \theta,$$

which simplifies to

$$mc^2(E_0 - E_i) = E_0E_i(1 - \cos \theta).$$

Hence

$$\lambda_i - \lambda_0 = \frac{h}{mc}(1 - \cos \theta).$$

The photon gains energy in this collision, so its wavelength decreases. Because both scattering angles are equal, the decrease in the first collision equals the increase in the second:

$$\boxed{\lambda_i = \lambda_f = 1.25 \times 10^{-10} \text{ m}.}$$

Initial electron speed.

All the first electron's initial kinetic energy is transferred to the photon:

$$K_{1e} = E_0 - E_i = hc \left(\frac{1}{\lambda_0} - \frac{1}{\lambda_i} \right).$$

Numerically,

$$K_{1e} = 1.54674 \times 10^{-17} \text{ J.}$$

Writing $E_{1e} = \gamma mc^2$, we have

$$\gamma = 1 + \frac{K_{1e}}{mc^2} = 1.0001888569.$$

Therefore

$$v_{1e} = c \sqrt{1 - \frac{1}{\gamma^2}} = 5.82963 \times 10^6 \text{ m s}^{-1},$$

so

$$\boxed{v_{1e} \approx 5.8 \times 10^6 \text{ m s}^{-1}.}$$

Since $v_{1e}/c \approx 0.0194$, the nonrelativistic approximation $K_{1e} \approx mv_{1e}^2/2$ would also be adequate at the precision of the supplied data.

Initial electron momentum.

Momentum conservation gives its direction and magnitude through

$$\begin{aligned} \vec{p}_{1e} &= \vec{p}_0 - \vec{p}_i, \\ p_{1e} &= h \sqrt{\frac{1}{\lambda_0^2} + \frac{1}{\lambda_i^2} - \frac{2 \cos 60^\circ}{\lambda_0 \lambda_i}}. \end{aligned}$$

Substituting the wavelengths,

$$p_{1e} = 5.30597 \times 10^{-24} \text{ kg m s}^{-1},$$

in agreement with $p_{1e} = \gamma mv_{1e}$. Thus

$$\boxed{p_{1e} \approx 5.3 \times 10^{-24} \text{ kg m s}^{-1}.}$$

Initial electron de Broglie wavelength.

The de Broglie relation is

$$\lambda_{\text{dB}} = \frac{h}{p_{1e}}.$$

Equivalently, using $\cos 60^\circ = \frac{1}{2}$,

$$\lambda_{\text{dB}} = \frac{\lambda_0 \lambda_i}{\sqrt{\lambda_0^2 + \lambda_i^2 - \lambda_0 \lambda_i}}.$$

Therefore

$$\lambda_{\text{dB}} = \frac{6.6 \times 10^{-34}}{5.30597 \times 10^{-24}} = 1.24388 \times 10^{-10} \text{ m},$$

or

$$\boxed{\lambda_{\text{dB}} \approx 1.24 \times 10^{-10} \text{ m}.}$$

The solution was checked against the official solution.

Problem 5: IPhO's logo

The Logo of the International Physics Olympiad is represented in the figure below.

The figure presents the phenomenon of the curving of the trajectory of a jet of fluid around the shape of a cylindrical surface. The trajectory of fluid is not like the expected dashed line but as the circular solid line.

Qualitatively explain this phenomenon (first observed by Romanian engineer Henry Coanda in 1936).

This problem will be not considered in the general score of the Olympiad. The best solution will be awarded a special prize.



Official English text of the problem.

Answers

- **Problem V.** This is the **Coandă effect**. The jet entrains surrounding fluid, and the cylinder restricts replacement of the fluid entrained from the space between the jet and the surface. The resulting lower pressure on the cylinder side produces an inward force, bending the jet around the cylinder.

Solution

Problem V.

Consider a jet passing approximately tangentially beside the cylinder. We neglect gravity over the region of the illustrated bend.

Through viscous shear—and turbulent mixing if the flow is turbulent—the jet transfers momentum to the surrounding fluid and carries some of it along. This process is called **entrainment**. For a free jet in otherwise symmetric surroundings, fluid can enter from both sides, so entrainment produces no preferred sideways deflection.

The nearby cylinder changes this situation. Fluid between the jet and the cylinder is entrained into the jet, but the solid surface restricts the supply of replacement fluid. Consequently, a region of reduced pressure develops on the cylinder side. The higher pressure on the exposed side then pushes the jet towards the cylinder.

Once the jet follows the curved surface, a pressure difference across it supplies the centripetal acceleration needed to maintain its curved trajectory. To express this quantitatively, let r measure distance from the cylinder axis, v be the local tangential speed, and ρ the fluid density. For locally circular flow, neglecting viscous forces in the normal momentum balance within the bulk of the jet,

$$-\frac{\partial p}{\partial r} = -\rho \frac{v^2}{r}, \quad \text{so} \quad \frac{\partial p}{\partial r} = \rho \frac{v^2}{r} > 0.$$

Thus pressure increases away from the cylinder: the pressure force points inward, exactly as required to turn the jet.

For example, for a thin jet of thickness $h \ll R$, approximately constant density, and representative bulk speed U , the required pressure difference is of order

$$p_{\text{outer}} - p_{\text{wall}} \sim \rho U^2 \frac{h}{R},$$

where R is the cylinder radius.

Entrainment establishes the asymmetric pressure field, and that pressure field bends the jet.

Attachment persists while the flow can maintain the pressure difference required by the curvature; if it cannot, the jet separates from the surface.

The solution was checked against the official solution.