

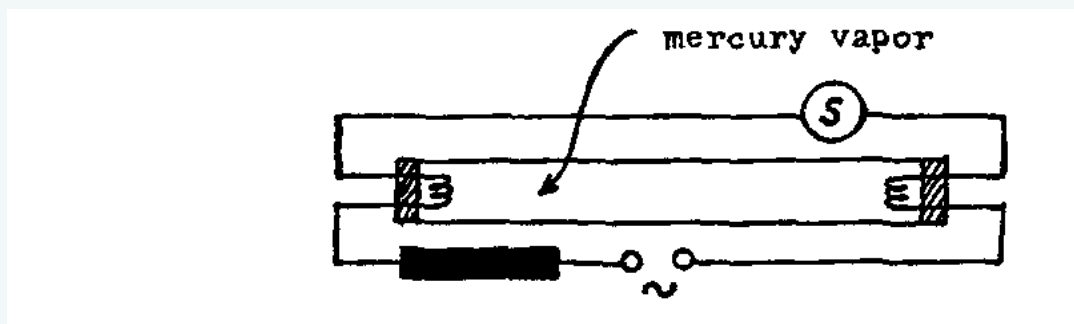
IPhO 1982

Theory problems and solutions · Malente, Federal Republic of Germany · 3 theory problems with complete solutions

eulersolve.org/ipro/1982/

Problem 1: Fluorescent lamp

An alternating voltage of 50 Hz frequency is applied to the fluorescent lamp shown in the accompanying circuit diagram.



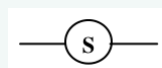
The following quantities are measured:

overall voltage (main voltage)	$U = 228.5 \text{ V}$
electric current	$I = 0.6 \text{ A}$
partial voltage across the fluorescent lamp	$U' = 84 \text{ V}$
ohmic resistance of the series reactor	$R_d = 26.3 \Omega$

The fluorescent lamp itself may be considered as an ohmic resistor in the calculations.

- What is the inductance L of the series reactor?
- What is the phase shift φ between voltage and current?
- What is the active power P_w transformed by the apparatus?
- Apart from limiting the current the series reactor has another important function. Name and explain this function!

Hint: The starter



includes a contact which closes shortly after switching on the lamp, opens up again and stays open.

- In a diagram with a quantitative time scale sketch the time sequence of the luminous flux emitted by the lamp.

- f) Why has the lamp to be ignited only once although the applied alternating voltage goes through zero in regular intervals?
- g) According to the statement of the manufacturer, for a fluorescent lamp of the described type a capacitor of about $4.7 \mu\text{F}$ can be switched in series with the series reactor. How does this affect the operation of the lamp and to what intent is this possibility provided for?
- h) Examine both halves of the displayed demonstrator lamp with the added spectroscope. Explain the differences between the two spectra. You may walk up to the lamp and you may keep the spectroscope as a souvenir.

Official English text of the problem.

Answers

- a) $L \approx 1.09 \text{ H}$.
- b) $\varphi \approx 64.1^\circ$, with the current **lagging** the mains voltage.
- c) $P_w \approx 59.9 \text{ W}$: 50.4 W in the lamp and 9.47 W in the reactor resistance.
- d) The reactor produces the **high-voltage ignition pulse** by self-induction when the starter opens after preheating the electrodes.
- e) The luminous flux pulsates at 100 Hz , repeating every 10 ms . An idealized sketch is $\Phi(t) \propto \sin^2(2\pi 50t - \varphi)$.
- f) Residual ionization and hot electrodes allow the discharge to resume after each brief current zero without another cold-start ignition pulse.
- g) The capacitor approximately preserves the current and mean brightness while making the current **lead** the voltage. For $4.7 \mu\text{F}$, $I_C \approx 0.611 \text{ A}$, leading by 63.6° . Pairing this lamp with an unmodified lamp gives nearly unity overall power factor and reduces flicker and stroboscopic effects.
- h) Assuming the demonstrator has uncoated and phosphor-coated halves: the uncoated half shows discrete mercury emission lines; the coated half additionally shows broad visible phosphor emission, produced by conversion of ultraviolet radiation into visible light.

Solution

The measured voltages and current are taken to be RMS values. During normal operation the starter is open. We use the stated ohmic approximation for the lamp and model the reactor as an ideal inductance in series with its winding resistance.

a) Inductance of the series reactor

The lamp resistance is

$$R_{\text{lamp}} = \frac{U'}{I} = \frac{84}{0.6} = 140 \Omega.$$

Thus the total series resistance is

$$R = R_{\text{lamp}} + R_d = 140 + 26.3 = 166.3 \Omega.$$

The resistive voltage is in phase with the current, while the inductive voltage leads the current by 90° . Consequently, the RMS voltages obey the phasor relation

$$U^2 = (IR)^2 + (I\omega L)^2, \quad \omega = 2\pi f.$$

The total resistive voltage is

$$IR = U' + IR_d = 84 + 0.6(26.3) = 99.78 \text{ V}.$$

Hence the inductive voltage is

$$U_L = \sqrt{228.5^2 - 99.78^2} = 205.56 \text{ V},$$

and

$$L = \frac{U_L}{2\pi f I} = \frac{205.56}{2\pi(50)(0.6)} = \boxed{1.09 \text{ H}}.$$

The corresponding inductive reactance is

$$X_L = \omega L = \boxed{342.6 \Omega}.$$

b) Phase shift

For a series resistance and inductance,

$$\tan \varphi = \frac{X_L}{R},$$

so

$$\varphi = \tan^{-1}\left(\frac{342.6}{166.3}\right) = \boxed{64.1^\circ}.$$

The current lags the applied voltage. Equivalently,

$$\cos \varphi = \frac{IR}{U} = \frac{99.78}{228.5} = 0.4367.$$

With the time origin at a rising zero crossing of the mains voltage,

$$u(t) = \sqrt{2} U \sin \omega t, \quad i(t) = \sqrt{2} I \sin(\omega t - \varphi).$$

c) Active power

An ideal inductance stores and returns energy, consuming no average power. The active power is therefore dissipated in the lamp and the winding resistance:

$$P_w = I^2(R_{\text{lamp}} + R_d).$$

Numerically,

$$P_w = 0.6^2(166.3) = 59.868 \text{ W} \approx \boxed{59.9 \text{ W}}.$$

Its two contributions are

$$P_{\text{lamp}} = U' I = 84(0.6) = \boxed{50.4 \text{ W}},$$

$$P_d = I^2 R_d = 0.6^2(26.3) = \boxed{9.47 \text{ W}}.$$

This agrees with $P_w = UI \cos \varphi$.

d) Ignition by self-induction

When the starter closes, current flows through the reactor, the two electrode filaments, and the starter contact. This heats the electrodes, facilitating electron emission.

When the starter subsequently opens, the reactor opposes the rapid decrease of current. Faraday's law gives its induced emf:

$$\mathcal{E}_L = -L \frac{di}{dt}.$$

A rapid interruption therefore produces a large voltage pulse across the lamp. This pulse initiates electrical breakdown of the gas and establishes the discharge. Energy stored in the reactor's magnetic field,

$$E_L = \frac{1}{2} Li^2,$$

is available during this transient.

Once the discharge is established, the starter remains open and the reactor limits the operating current. The magnitude of the ignition pulse cannot be calculated from the supplied data because the interruption time and transient circuit properties are unspecified.

e) Time dependence of the luminous flux

For an idealized sketch, assume constant luminous efficacy and neglect the phosphor's emission delay. The luminous flux is then proportional to the instantaneous power delivered to the lamp:

$$p_{\text{lamp}}(t) = R_{\text{lamp}} i^2(t) = 2R_{\text{lamp}} I^2 \sin^2(\omega t - \varphi).$$

Here,

$$p_{\text{lamp}}(t) = (100.8 \text{ W}) \sin^2(\omega t - \varphi).$$

Thus

$$\boxed{\Phi(t) = \Phi_{\text{max}} \sin^2(\omega t - \varphi)} = \frac{\Phi_{\text{max}}}{2} [1 - \cos(2\omega t - 2\varphi)].$$

Both current directions produce light, so the luminous-flux period is

$$T_{\Phi} = \frac{1}{2f} = \boxed{10 \text{ ms}},$$

corresponding to $\boxed{100 \text{ Hz}}$.

For the sketch, choose a local time $\tau = 0$ at a current zero. Draw a smooth, nonnegative curve through these points and repeat it every 10 ms:

τ (ms)	0	2.5	5.0	7.5	10.0
Φ/Φ_{max}	0	1/2	1	1/2	0

The curve has horizontal tangents at its minima and maxima. Its mean is $\Phi_{\text{max}}/2$.

Relative to the mains-voltage time origin used in part (b), the first current zero occurs at

$$t = \frac{\varphi}{\omega} = 3.56 \text{ ms}.$$

Accordingly, ideal light minima occur at 3.56, 13.56, ... ms, and maxima at 8.56, 18.56, ... ms.

The absolute luminous flux is undetermined because no luminous efficacy is supplied. In a real lamp, phosphor persistence smooths and delays the light output, so its minima need not reach zero.

f) Why a fresh ignition pulse is unnecessary

The large starting voltage is needed to establish a discharge in the initially weakly ionized gas. During operation, the gas contains charged particles and the electrodes remain hot.

Around each current zero, the interruption is too brief for complete loss of ionization and cooling of the electrodes. The discharge can therefore resume in the next half-cycle at a much lower voltage than was required for the initial cold start.

Also, the source-voltage zeros and current zeros do not coincide because of the phase shift. Nevertheless, the current does cross zero every 10 ms; residual ionization and the heated electrodes explain why another starter pulse is unnecessary.

g) Effect and purpose of the series capacitor

With a capacitor in series, the impedance becomes

$$Z_C = R + j \left(\omega L - \frac{1}{\omega C} \right).$$

For $C = 4.7 \mu\text{F}$,

$$X_C = \frac{1}{2\pi(50)(4.7 \times 10^{-6})} = 677.3 \Omega.$$

The net reactance is therefore

$$X_{\text{net}} = 342.6 - 677.3 = -334.6 \Omega.$$

It is capacitive and has almost the same magnitude as the original inductive reactance.

The new current is

$$I_C = \frac{U}{\sqrt{R^2 + X_{\text{net}}^2}} = \frac{228.5}{\sqrt{166.3^2 + 334.6^2}} = \boxed{0.611 \text{ A}}.$$

The impedance phase is

$$\varphi_C = \tan^{-1} \left(\frac{-334.6}{166.3} \right) = -63.6^\circ,$$

so the current now **leads** the voltage by $\boxed{63.6^\circ}$.

The lamp power becomes

$$P_{\text{lamp},C} = I_C^2 R_{\text{lamp}} \approx 52.3 \text{ W},$$

close to its original 50.4 W. Thus its mean brightness is approximately preserved.

To see why this capacitance was chosen, require the current magnitude to remain exactly unchanged:

$$(X_L - X_C)^2 = X_L^2.$$

The nonzero solution is

$$X_C = 2X_L,$$

giving

$$C_* = \frac{1}{2\omega X_L} = \frac{1}{2\omega^2 L} = \boxed{4.65 \mu\text{F}},$$

which agrees with the manufacturer's approximate $4.7 \mu\text{F}$. At this value, the impedance changes from $R + jX_L$ to $R - jX_L$.

This permits a useful **two-lamp arrangement**: connect two lamp branches in parallel across the supply, one with the original reactor and one with the reactor plus series capacitor.

- **Power-factor improvement.** For the ideal equal-current case, the two currents lag and lead by equal angles. Their reactive components cancel:

$$I_{\text{total}} = \frac{U}{R + jX_L} + \frac{U}{R - jX_L} = \frac{2UR}{R^2 + X_L^2}.$$

The total current is in phase with the voltage, giving unity power factor. With the nominal $4.7 \mu\text{F}$, cancellation is nearly complete.

- **Reduced light modulation.** The lamps reach their light minima at different times. Under the model of part (e), their combined flux is

$$\begin{aligned}\Phi_{\text{pair}}(t) &= \Phi_{\text{max}} [\sin^2(\omega t - \varphi) + \sin^2(\omega t + \varphi)] \\ &= \Phi_{\text{max}} [1 - \cos(2\varphi) \cos(2\omega t)].\end{aligned}$$

Since $|\cos(2\varphi)| \approx 0.619 < 1$, the combined light never falls to zero in this model. Flicker and stroboscopic effects are reduced, although the light is not perfectly constant.

h) Spectra of the two halves

The expected interpretation is that the demonstrator has one uncoated half and one half coated with fluorescent phosphor.

In the **uncoated half**, the observed visible light comes directly from the mercury discharge. Excited mercury atoms emit photons at discrete transition energies,

$$h\nu = E_{\text{upper}} - E_{\text{lower}},$$

so the spectroscope shows separated emission lines. Much of the discharge radiation is ultraviolet and is therefore invisible to the eye.

In the **phosphor-coated half**, the coating absorbs ultraviolet radiation and re-emits part of its energy as visible light. This fluorescence produces broad visible emission bands, often appearing nearly continuous in a simple spectroscope. Direct mercury lines can remain superposed on this broader emission.

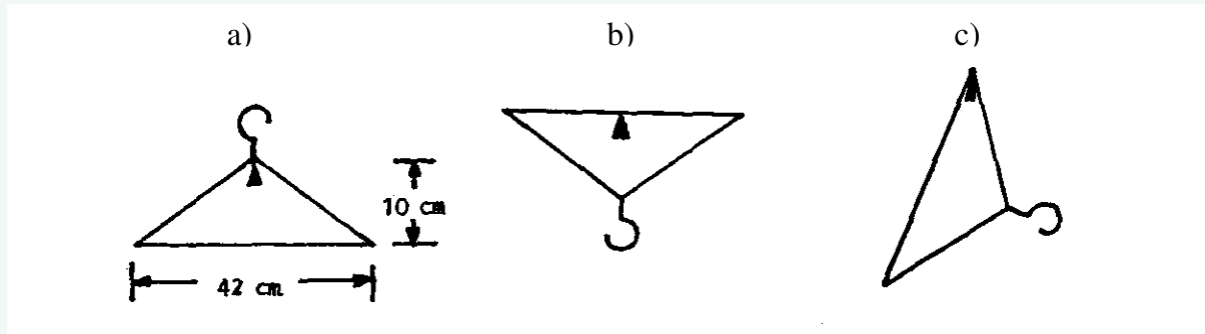
The coating therefore increases the useful visible output and changes its colour toward white. The spectral difference results from ultraviolet-to-visible conversion in the phosphor.

The solution was checked against the official solution.

Problem 2: Oscillating coat hanger

A (suitably made) wire coat hanger can perform small amplitude oscillations in the plane of the figure around the equilibrium positions shown. In positions a) and b) the long side is horizontal. The other two sides have equal length. The period of oscillation is the same in all cases.

What is the location of the center of mass, and how long is the period?



The figure does not contain any information beyond the dimensions given. Nothing is known, e.g., concerning the detailed distribution of mass.

Official English text of the problem.

Answers

- **Center of mass:** Halfway along the triangle's altitude, on its symmetry axis: 5.0 cm from the long side and 5.0 cm from the opposite vertex.
- **Common period:**

$$T = 2\pi \sqrt{\frac{0.050 + \sqrt{0.210^2 + 0.050^2}}{g}} \simeq 1.03 \text{ s},$$

where the lengths are in metres and $g = 9.81 \text{ m s}^{-2}$.

Solution

Treat the hanger as a rigid physical pendulum, with a fixed suspension point in each configuration and negligible friction. Let its mass be M , and let $I_G = Mk^2$ be its moment of inertia about the axis through its center of mass perpendicular to its plane.

If the center of mass lies a distance d below the suspension point, the parallel-axis theorem and gravitational torque give

$$M(k^2 + d^2)\ddot{\theta} = -Mgd \sin \theta.$$

For small oscillations, $\sin \theta \simeq \theta$, so

$$T = 2\pi \sqrt{\frac{k^2 + d^2}{gd}}.$$

Thus a common period means that

$$\ell \equiv \frac{gT^2}{4\pi^2} = d + \frac{k^2}{d}$$

has the same value for all three suspension points.

Location of the center of mass.

Denote the vertex opposite the long side by A , the midpoint of the long side by B , and the corner used for suspension in c) by C . The given dimensions are

$$AB = h = 0.100 \text{ m}, \quad BC = a = 0.210 \text{ m}.$$

In position a), equilibrium requires the center of mass G to lie vertically below A . Since the triangle is isosceles and its long side is horizontal, G lies on the line AB . In position b), the suspension point must likewise lie vertically above G , so it is B .

Both equilibria must be stable. Consequently, G lies between A and B . Writing $BG = x$, we have

$$0 < x < h, \quad d_a = h - x, \quad d_b = x, \quad d_c = \sqrt{a^2 + x^2}.$$

Equality of the periods in a) and b) gives

$$h - x + \frac{k^2}{h - x} = x + \frac{k^2}{x},$$

or

$$(h - 2x)[x(h - x) - k^2] = 0.$$

Hence either

$$x = \frac{h}{2} \quad \text{or} \quad k^2 = x(h - x).$$

The second possibility would imply

$$\ell = x + \frac{x(h - x)}{x} = h.$$

But in configuration c),

$$\ell = d_c + \frac{k^2}{d_c} > d_c \geq a = 0.210 \text{ m} > h.$$

This contradicts the common period. Therefore

$$\boxed{x = \frac{h}{2} = 0.050 \text{ m}.$$

The center of mass is thus halfway between the long side's midpoint and the opposite vertex. This conclusion does not require a uniform mass distribution.

Period of oscillation.

With this center of mass, the suspension distances are

$$d_a = d_b = d = 0.050 \text{ m}, \quad d_c = r = \sqrt{0.210^2 + 0.050^2} = 0.215870 \dots \text{ m}.$$

Equating the periods for distances d and r ,

$$d + \frac{k^2}{d} = r + \frac{k^2}{r}.$$

Since $d \neq r$, this yields

$$k^2 = dr.$$

The common equivalent pendulum length is therefore

$$\ell = d + \frac{dr}{d} = d + r = 0.265870 \dots \text{ m}.$$

Finally,

$$T = 2\pi \sqrt{\frac{d+r}{g}} = 2\pi \sqrt{\frac{0.265870 \dots}{9.81}} \simeq 1.03 \text{ s}.$$

The solution was checked against the official solution.

Problem 3: Hot-air-balloon

Consider a hot-air balloon with fixed volume $V_B = 1.1 \text{ m}^3$. The mass of the balloon-envelope, whose volume is to be neglected in comparison to V_B , is $m_H = 0.187 \text{ kg}$.

The balloon shall be started, where the external air temperature is $\vartheta_1 = 20^\circ \text{C}$ and the normal external air pressure is $p_0 = 1.013 \cdot 10^5 \text{ Pa}$. Under these conditions the density of air is $\rho_1 = 1.2 \text{ kg/m}^3$.

a) What temperature ϑ_2 must the warmed air inside the balloon have to make the balloon just float?

b) First the balloon is held fast to the ground and the internal air is heated to a steady-state temperature of $\vartheta_3 = 110^\circ \text{C}$. The balloon is fastened with a rope.

Calculate the force on the rope.

c) Consider the balloon being tied up at the bottom (the density of the internal air stays constant). With a steady-state temperature $\vartheta_3 = 110^\circ \text{C}$ of the internal air the balloon rises in an isothermal atmosphere of 20°C and a ground pressure of $p_0 = 1.013 \cdot 10^5 \text{ Pa}$. Which height h can be gained by the balloon under these conditions?

d) At the height h the balloon (question c)) is pulled out of its equilibrium position by 10 m and then is released again.

Find out by qualitative reasoning what kind of motion it is going to perform!

Official English text of the problem.

Answers

- **a)** $\vartheta_2 \approx 68.4^\circ \text{C}$, corresponding to $T_2 \approx 341.5 \text{ K}$.
- **b)** Rope tension: $F_{\text{rope}} \approx 1.21 \text{ N}$. The balloon pulls upward on the rope.

- **c)** Equilibrium height above the starting point: $h \approx 8.4 \times 10^2 \text{ m} = 0.84 \text{ km}$.
- **d)** Approximately simple harmonic vertical oscillations about h , with amplitude 10 m when air resistance is neglected. Air resistance damps the motion.

Solution

Treat air as an ideal gas with constant composition,

$$p = \rho R_s T,$$

where R_s is its specific gas constant and T is absolute temperature. The relevant temperatures are

$$T_1 = 293.15 \text{ K}, \quad T_3 = 383.15 \text{ K}.$$

In (a) and (b), the balloon is open at the bottom, so its internal pressure equals the surrounding pressure. We neglect pressure and density variations over the balloon's dimensions. After sealing in (c), its enclosed air mass and prescribed volume remain constant.

Use $g = 9.81 \text{ m s}^{-2}$, treated as constant over the heights involved.

a) Temperature required to just float

By Archimedes' principle, the upward buoyant force equals the weight of the displaced external air:

$$F_B = \rho_1 V_B g.$$

For neutral buoyancy, this balances the combined weight of the envelope and the enclosed air:

$$\rho_1 V_B g = (m_H + \rho_2 V_B) g.$$

Thus the required internal density is

$$\rho_2 = \rho_1 - \frac{m_H}{V_B} = 1.2 - \frac{0.187}{1.1} = 1.03 \text{ kg m}^{-3}.$$

Since the internal and external pressures are equal, the ideal-gas law gives

$$\rho_2 T_2 = \rho_1 T_1.$$

Therefore

$$T_2 = \frac{\rho_1 T_1}{\rho_2} = \frac{1.2(293.15)}{1.03} = 341.53 \text{ K},$$

or

$$\vartheta_2 = 341.53 - 273.15 \approx 68.4 \text{ }^\circ\text{C}.$$

b) Force on the rope

At $T_3 = 383.15 \text{ K}$, the internal density is

$$\rho_3 = \rho_1 \frac{T_1}{T_3} = 1.2 \frac{293.15}{383.15} \approx 0.91813 \text{ kg m}^{-3}.$$

The rope supplies the downward force needed to balance the excess of buoyancy over weight:

$$F_{\text{rope}} = \rho_1 V_B g - (m_H + \rho_3 V_B) g.$$

Substituting,

$$F_{\text{rope}} = [(1.2 - 0.91813)(1.1) - 0.187](9.81),$$

so

$$F_{\text{rope}} \approx 1.21 \text{ N}.$$

This is the tension: the rope pulls downward on the balloon, and the balloon pulls upward on the rope.

c) Equilibrium height in the isothermal atmosphere

Once the bottom is tied shut, the balloon's total mass remains

$$M = m_H + \rho_3 V_B = 0.187 + (0.91813)(1.1) \approx 1.19694 \text{ kg}.$$

To find how buoyancy changes with height, combine atmospheric hydrostatic equilibrium,

$$\frac{dp}{dz} = -\rho g,$$

with the ideal-gas law at the constant external temperature T_1 :

$$\frac{\rho(z)}{p(z)} = \frac{\rho_1}{p_0}.$$

Consequently,

$$\frac{dp}{p} = -\frac{\rho_1 g}{p_0} dz.$$

Integrating from ground level, where $p = p_0$, gives

$$p(z) = p_0 e^{-z/H}, \quad \rho(z) = \rho_1 e^{-z/H},$$

where the atmospheric scale height is

$$H = \frac{p_0}{\rho_1 g} = \frac{1.013 \times 10^5}{(1.2)(9.81)} \approx 8.605 \times 10^3 \text{ m}.$$

At equilibrium, buoyancy equals the balloon's constant weight:

$$\rho(h) V_B g = M g.$$

The external density there must therefore be

$$\rho(h) = \frac{M}{V_B} = \rho_3 + \frac{m_H}{V_B} \approx 1.08813 \text{ kg m}^{-3}.$$

Using the exponential density profile,

$$\rho_1 e^{-h/H} = \rho_3 + \frac{m_H}{V_B},$$

and hence

$$h = H \ln \left(\frac{\rho_1}{\rho_3 + m_H/V_B} \right).$$

Numerically,

$$h = (8605.16) \ln \left(\frac{1.2}{1.08813} \right) \approx 842 \text{ m}.$$

Thus

$$h \approx 0.84 \text{ km}.$$

This is the equilibrium height identified in part (d). A freely rising balloon can pass through it because of its inertia.

d) Motion after displacement

The balloon's weight and volume remain constant, whereas external air density decreases with height.

- If displaced **upward**, it encounters less dense air. Buoyancy becomes smaller than its weight, so the resultant force is downward.
- If displaced **downward**, it encounters denser air. Buoyancy becomes greater than its weight, so the resultant force is upward.

The force therefore restores the balloon toward h . On reaching h , its inertia carries it through equilibrium, leading to oscillation.

The motion is approximately harmonic because the displacement is small compared with the atmospheric scale height. To see this, let $x = z - h$. Since $\rho(h)V_B = M$, the upward resultant force is

$$F(x) = Mg \left(e^{-x/H} - 1 \right).$$

Here

$$\frac{|x|}{H} \leq \frac{10}{8605} \approx 1.2 \times 10^{-3} \ll 1,$$

so

$$F(x) \approx -\frac{Mg}{H}x.$$

This is a restoring force proportional to displacement.

Thus, neglecting air resistance, the balloon performs **approximately simple harmonic vertical oscillations**, initially of amplitude 10 m. Air resistance dissipates energy and damps the motion, eventually bringing the balloon to rest at its equilibrium height.

The solution was checked against the official solution.