

IPhO 1967

Theory problems and solutions · Warsaw, Poland · 4 theory problems with complete solutions

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Problem 1: Bullet passing through a ball resting on a column

A small ball with mass $M = 0.2 \text{ kg}$ rests on a vertical column with height $h = 5 \text{ m}$. A bullet with mass $m = 0.01 \text{ kg}$, moving with velocity $v_0 = 500 \text{ m/s}$, passes horizontally through the center of the ball (Fig. 1). The ball reaches the ground at a distance $s = 20 \text{ m}$. Where does the bullet reach the ground? What part of the kinetic energy of the bullet was converted into heat when the bullet passed through the ball? Neglect resistance of the air. Assume that $g = 10 \text{ m/s}^2$.

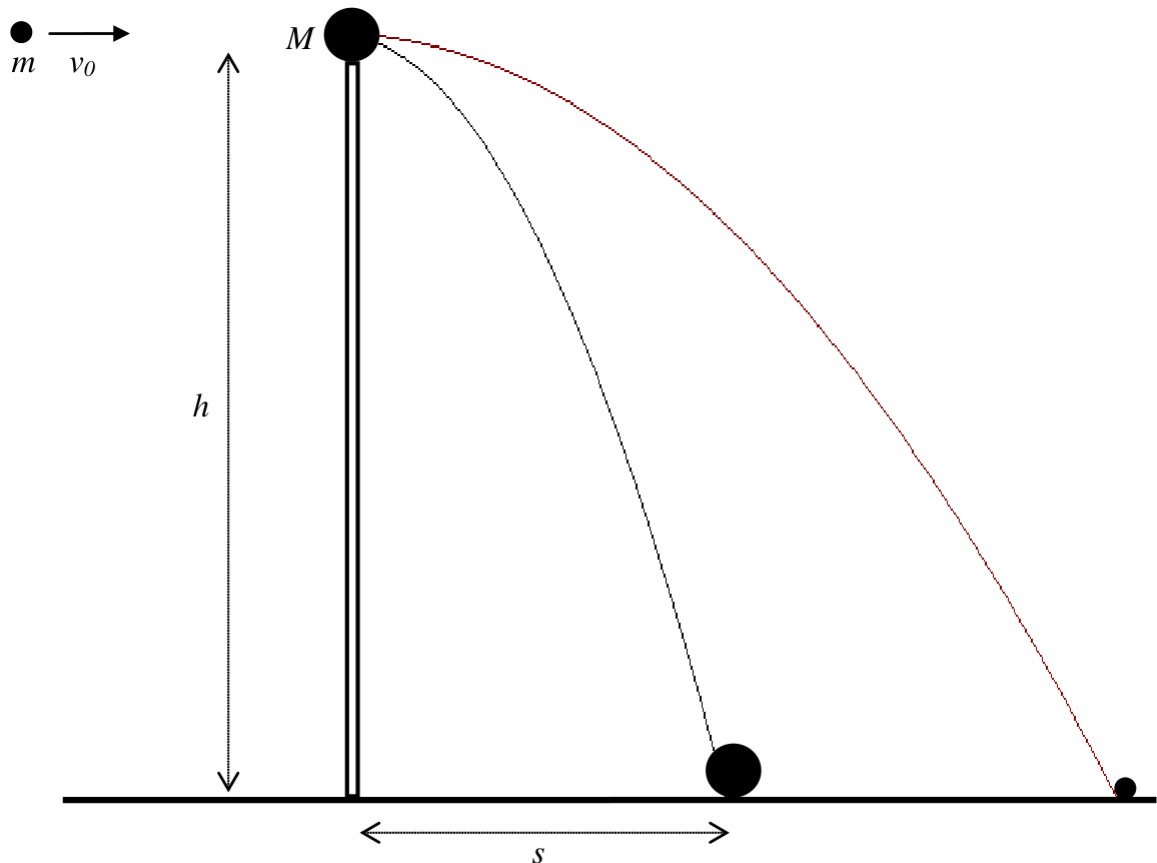


Fig. 1

Official English text of the problem.

Answers

- **Bullet's landing point:** 100 m horizontally from the foot of the column, in the bullet's original direction of motion—80 m beyond the ball.

- **Energy converted into heat:** $Q = 1160 \text{ J}$, corresponding to 92.8% of the bullet's initial kinetic energy.

Solution

Bullet's landing point

Treat the ball and bullet as particles and take the penetration time to be negligible compared with the subsequent fall time. The central, horizontal impact produces horizontal translational motion without rotation. During the brief impact, the external horizontal impulse is negligible, so horizontal momentum is conserved.

After penetration, both objects start falling from height h with zero vertical velocity. Their common fall time is therefore

$$h = \frac{1}{2}gt^2, \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(5)}{10}} = 1 \text{ s}.$$

With no air resistance, the ball's horizontal velocity V remains constant. Its measured range gives

$$V = \frac{s}{t} = \frac{20}{1} = 20 \text{ m s}^{-1}.$$

Let v be the bullet's horizontal velocity after passing through the ball. Conservation of horizontal momentum gives

$$mv_0 = mv + MV,$$

hence

$$v = v_0 - \frac{M}{m}V = 500 - \frac{0.2}{0.01}(20) = 100 \text{ m s}^{-1}.$$

The bullet's horizontal range from the column is thus

$$x = vt = (100)(1) = 100 \text{ m}.$$

Energy converted into heat

Kinetic energy is not conserved during penetration. In the problem's idealized model, the decrease in total kinetic energy is converted into heat. The change in gravitational potential energy during the brief penetration is negligible.

Initially, only the bullet has kinetic energy:

$$K_i = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.01)(500)^2 = 1250 \text{ J}.$$

Immediately after penetration, the total kinetic energy is

$$K_f = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}(0.01)(100)^2 + \frac{1}{2}(0.2)(20)^2 = 50 + 40 = 90 \text{ J}.$$

Therefore,

$$Q = K_i - K_f = 1250 - 90 = 1160 \text{ J}.$$

The required fraction of the bullet's initial kinetic energy is

$$\frac{Q}{K_i} = \frac{1160}{1250} = 0.928 = 92.8\%.$$

The 40 J transferred to the ball's motion must be excluded from the heat.

The solution was checked against the official solution.

Problem 2: Resistance of an infinite resistor network

Consider an infinite network consisting of resistors (resistance of each of them is r) shown in Fig. 3. Find the resultant resistance R_{AB} between points A and B.

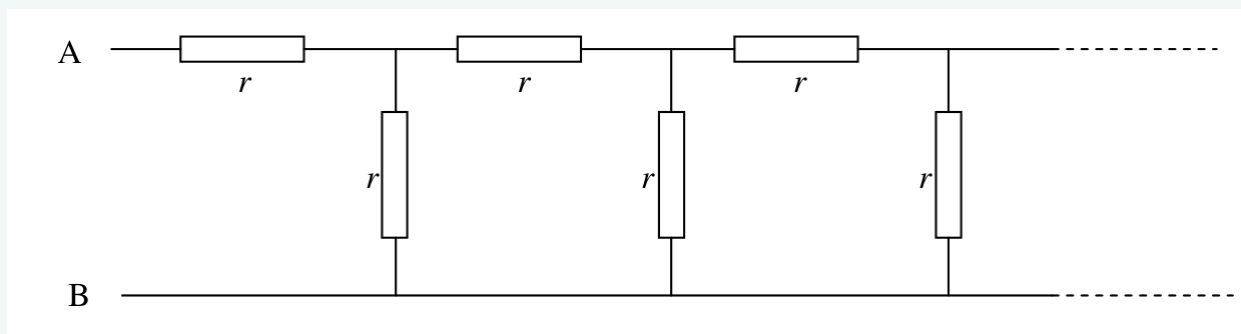


Fig. 3

Official English text of the problem.

Answers

• **Problem 2:** $R_{AB} = \frac{1 + \sqrt{5}}{2} r \approx 1.618 r.$

Solution

Treat the connecting wires as having negligible resistance and each resistor as an ideal ohmic resistor with $r > 0$.

Let $R = R_{AB}$. Current entering at A must first pass through the first horizontal resistor, of resistance r . At the following junction, there are two paths to the lower wire connected to B:

- The first vertical resistor, of resistance r .
- The entire remaining ladder, which has exactly the same structure as the original network and therefore has resistance R .

These two paths are in parallel, and their equivalent resistance is in series with the first horizontal resistor. The series and parallel resistance laws therefore give

$$R = r + \left(\frac{1}{r} + \frac{1}{R} \right)^{-1} = r + \frac{rR}{r + R}.$$

Multiplying by $r + R$ and rearranging,

$$R(r + R) = r(r + R) + rR,$$

$$R^2 - rR - r^2 = 0.$$

Thus

$$R = \frac{r \pm \sqrt{r^2 + 4r^2}}{2} = \frac{1 \pm \sqrt{5}}{2} r.$$

The negative root is inadmissible for a network of positive resistances, leaving

$$R_{AB} = \frac{1 + \sqrt{5}}{2} r \approx 1.618 r.$$

To justify taking an infinite ladder, let R_n denote the resistance of a finite ladder ending after its n th vertical resistor. Then

$$R_1 = 2r, \quad R_{n+1} = r + \frac{rR_n}{r + R_n}.$$

Here $R_2 = 5r/3 < R_1$. Since the right-hand side is an increasing function of R_n , induction shows that $R_{n+1} < R_n$ for every n . Also, $R_n > r$, because the first horizontal resistor is in series with a positive resistance. Hence R_n converges, and its limit satisfies the equation solved above.

The solution was checked against the official solution.

Problem 3: Equal heat supplied to a hanging ball and a ball resting on a plane

Consider two identical homogeneous balls, A and B, with the same initial temperatures. One of them is at rest on a horizontal plane, while the second one hangs on a thread (Fig. 6). The same quantities of heat have been supplied to both balls. Are the final temperatures of the balls the same or not? Justify your answer. (All kinds of heat losses are negligible.)

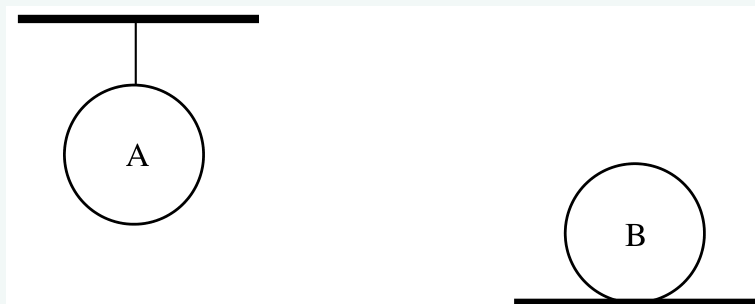


Fig. 6

Official English text of the problem.

Answers

- **Problem 3:** In the usual elementary model, assuming positive thermal expansion, **the hanging ball A becomes hotter than the resting ball B:**

$$T_A > T_B.$$

Solution

Assume that the plane is rigid, the thread has a fixed length, and both balls expand uniformly when heated. We use the elementary approximation that their thermal-energy changes have the same dependence on temperature, neglecting the influence of supporting stresses.

Let each ball initially have radius R_0 and temperature T_0 . Denote their common mass by m and the heat supplied to each by Q .

When the radius increases by ΔR :

- For **A**, the top attachment point stays fixed, so its centre of mass **descends** by ΔR .
- For **B**, its lowest point remains on the plane, so its centre of mass **rises** by ΔR .

Their gravitational potential-energy changes are therefore

$$\Delta E_{g,A} = -mg\Delta R_A, \quad \Delta E_{g,B} = +mg\Delta R_B.$$

The stationary supports do no work. Conservation of energy gives

$$Q = \Delta U + \Delta E_g,$$

where common work against atmospheric pressure may either be neglected or included in the thermal-energy term. Thus

$$\Delta U_A = Q + mg\Delta R_A, \quad \Delta U_B = Q - mg\Delta R_B.$$

For positive thermal expansion, both radius changes are positive. Consequently,

$$\Delta U_A > Q > \Delta U_B.$$

With the assumed common, increasing relationship between thermal energy and temperature, this implies

$$T_A > T_B.$$

Explicitly, let c be the common specific heat capacity and $\alpha > 0$ the linear expansion coefficient, treated as constant over a small temperature interval. Writing $\Delta T_i = T_i - T_0$,

$$\Delta U_i = mc\Delta T_i, \quad \Delta R_i = \alpha R_0 \Delta T_i.$$

The energy balances become

$$Q = m(c - g\alpha R_0)\Delta T_A, \quad Q = m(c + g\alpha R_0)\Delta T_B,$$

and hence

$$T_A = T_0 + \frac{Q}{m(c - g\alpha R_0)}, \quad T_B = T_0 + \frac{Q}{m(c + g\alpha R_0)}.$$

For ordinary conditions, $g\alpha R_0 \ll c$, giving the stated ordering.

Physically, A releases gravitational potential energy as it expands, whereas B uses some supplied energy to raise its centre of mass. No numerical temperature difference can be calculated from the supplied information.

The solution was checked against the official solution.

Problem 4: Pressure in a closed vessel with air and evaporated water

Comment: The Organizing Committee prepared three theoretical problems. Unfortunately, at the time of the 1st Olympiad the Romanian students from the last class had the entrance examinations at the universities. For that Romania sent a team consisting of students from younger classes. They were not familiar with electricity. To give them a chance the Organizers (under agreement of the International Board) added the fourth problem presented here. The students (not only from Romania) were allowed to choose three problems. The maximum possible scores for the problems were: 1st problem – 10 points, 2nd problem – 10 points, 3rd problem – 10 points and 4th problem – 6 points. The fourth problem was solved by 8 students. Only four of them solved the problem for 6 points.

A closed vessel with volume $V_0 = 10 \text{ l}$ contains dry air in the normal conditions ($t_0 = 0^\circ\text{C}$, $p_0 = 1 \text{ atm}$). In some moment 3 g of water were added to the vessel and the system was warmed up to $t = 100^\circ\text{C}$. Find the pressure in the vessel. Discuss assumption you made to solve the problem.

Official English text of the problem.

Answers

- **Problem 4 (6 points):** The final absolute pressure is

$$p \simeq 1.88 \text{ atm} \simeq 1.90 \times 10^5 \text{ Pa.}$$

All 3 g of water evaporate; the resulting vapour is unsaturated. This assumes a rigid vessel, no loss of air during water addition, ideal-gas behaviour, and thermal and phase equilibrium at 100°C .

Solution

Problem 4

Use absolute temperatures:

$$T_0 = 273.15 \text{ K}, \quad T = 373.15 \text{ K}, \quad V_0 = 1.00 \times 10^{-2} \text{ m}^3.$$

For an ideal mixture, Dalton's law gives the total pressure as the sum of the air and water-vapour partial pressures:

$$p = p_a + p_v.$$

The amount of air is fixed. Applying the ideal-gas law initially and finally,

$$n_a = \frac{p_0 V_0}{RT_0}, \quad p_a = \frac{n_a RT}{V_0} = p_0 \frac{T}{T_0}.$$

Thus

$$p_a = 1 \text{ atm} \times \frac{373.15}{273.15} \simeq 1.366 \text{ atm}.$$

We must determine whether all the added water evaporates. If it does, its amount is

$$n_v = \frac{m}{M} = \frac{3.0 \text{ g}}{18.0 \text{ g mol}^{-1}} = \frac{1}{6} \text{ mol}.$$

Its corresponding partial pressure would be

$$p_v^{(\text{all})} = \frac{mRT}{MV_0} = \frac{(1/6)(8.314)(373.15)}{1.00 \times 10^{-2}} \simeq 5.17 \times 10^4 \text{ Pa} \simeq 0.510 \text{ atm}.$$

At 100°C , water's saturation vapour pressure is approximately

$$p_{\text{sat}} = 1 \text{ atm}.$$

Since

$$p_v^{(\text{all})} < p_{\text{sat}},$$

even complete evaporation produces unsaturated vapour. Any liquid remaining would therefore continue to evaporate. Consequently, **all the water evaporates**.

Equivalently, the water mass needed just to saturate the vessel is

$$m_{\text{sat}} = \frac{Mp_{\text{sat}}V_0}{RT} \simeq 5.88 \text{ g},$$

which exceeds the available 3 g. Saturation is determined by the water's **partial pressure**.

The final total pressure is therefore

$$p = p_0 \frac{T}{T_0} + \frac{mRT}{MV_0} \simeq 1.366 + 0.510 = 1.876 \text{ atm} \simeq 1.90 \times 10^5 \text{ Pa}.$$

The assumptions underlying this result are:

- **Constant vessel volume:** thermal expansion and pressure-induced deformation of the vessel are negligible.
- **Conservation of air:** adding the water does not allow air to escape, and the vessel remains sealed.
- **Ideal-gas behaviour:** air and water vapour obey the ideal-gas law and Dalton's law. These are reasonable approximations at the stated temperature and pressures.
- **Equilibrium:** sufficient heat and time are supplied for the entire contents to reach 100°C and for evaporation to finish. The latent heat of evaporation affects the required heat input, but not the final pressure at the specified temperature.

The solution was checked against the official solution.